Eight-directions fuzzy asymmetric division and analysis of its uncertainty conduced by positioning error of reference point

GUO Jifa[1,2], **CUI Weihong**[2]

1. *Graduate University of the Chinese Academy of Sciences*, *Beijing* 100049, *China*;
2. *Institute of Remote Sensing Applications*, *Chinese Academy of Sciences*, *Beijing* 100101, *China*

Abstract: It is necessary to consider the uncertainty of spatial data and fuzziness of relative conception when discussing the description of spatial relation and reasoning because of the complexity of direction relation induced by the fuzziness of direction concept and inherent uncertainty of spatial data. Two fuzzy models are introduced based on classical fuzzy set. In 4-directions fuzzy model the space is divided into four cardinal directions and each direction has equal angle, but each main cardinal direction has 60° and each secondary cardinal direction has 30° in 8- directions fuzzy asymmetric model. Extended 8-directions fuzzy asymmetric model is introduced based on interval type-2 fuzzy sets which takes the positioning error of reference point into account. The primary membership function and the uncertainty of primary membership grade is discussed too. The difference between this model and the cone-based model is comparatively analyzed. Two cases are provided in the last. The first case is used to analysis the attributes of 8- directions fuzzy asymmetric model and the second case shows the process of determining the direction relation between point with positioning error and polygon.

Key words: cardinal direction relation, interval type-2 fuzzy set, direction membership grade error

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1 INTRODUCTION

Qualitative direction relation representation and reasoning is important in geographical information science and artificial intelligence, and plentiful harvest have been producted in this field. Nowadays the existing direction models are cone-based model, minimum enclosing rectangle model (MBR), 2-D string model, Freksa-Zimmermann model, direction relation matrix model and voronio model (Guo *et al.*, 2006; Du *et al.*, 2007). Fuzzy set theory, rough set theory and probability theory were used in qualitative direction relation analysis by many researchers (Papadias *et al.*, 1999; Du *et al.*, 2004, 2005; Cao *et al.*, 2001; Goyal, 2000; Du and Wang, 2004; Liu, 2008). Qualitative direction relation representation and reasoning were affected by many factors. Lots of faults exist in this field:

(1) Some models are not fuzzy and not consisted with human concept, but the direction concepts are fuzziness, the test finished by Jin *et al.* (2009) has proved it.

(2) Some models like cone-based model, MBR divide the space by crisp style, and the translation between direction tiles are crisp. Some papers use an interval band as the translation band or fuzzy interval, and this is a subjective method.

(3) The membership grade in inner of tile can't be expressed very well.

(4) The membership functions for each tile are invalid. The membership value should have error since the inherent positioning error of spatial data. This type of error should have relation with distance between reference object and target object. However it was not discussed until now.

(5) Representation of direction relation between geographical objects wasn't coincident with human concept in existing models.

The 8-direction fuzzy asymmetric model (DFAM) is introduced based on fuzzy set theory when the positioning error of reference point wasn't taken into account. Then the extended 8-direnctions fuzzy asymmetric model (EDFAM) is proposed based on interval type-2 fuzzy set, which can express the membership grade error and use that type of error in reasoning. The process of building primary membership function is discussed in detail, and the attributes of EDMF are drastic analyzed.

2 FUZZY DIVISION OF SPACE WHILE DON'T CONSIDE POSITIONING ERROR OF REFERENCT POINT

The existing 4-directions cone-based model has little con-

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First author biography: GUO Jifa (1981—), male, Ph.D, lecture, graduated from Graduate University of Chinese Academy of Sciences in 2010. His research focuses on spatio-temporal database, spatial data uncertainty and quality. E-mail: guojfx2004@163.com

nection with 8-directions model. 8-directions models are based on 4-directions models as human direction concept, so the space can be divided into 16 tiles or 32 tiles.

2.1 4-directions fuzzy division

The 4-directions fuzzy partition method is showed as Fig.1(a) (Guo *et al*., 2006). Membership functions of each tile are showed as Fig.1(b), the membership function of north (N) can expressed as Eq. (1), and membership functions of direction east (E), south (S), west (W) can built with the same method.

$$\mu_{\mathrm{N}}(\theta)=\begin{cases}-\dfrac{2}{\pi}\left(\theta-\dfrac{\pi}{2}\right), & 0\leqslant\theta\leqslant\dfrac{\pi}{2}\\ 0, & \dfrac{\pi}{2}<\theta\leqslant\dfrac{3}{2}\pi\\ \dfrac{2}{\pi}\left(\theta-\dfrac{3}{2}\pi\right), & \dfrac{3}{2}\pi<\theta\leqslant 2\pi\end{cases}\tag{1}$$

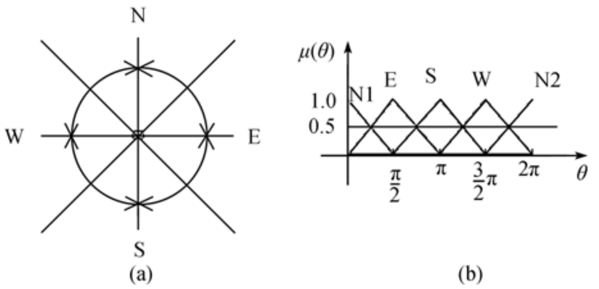

Fig. 1 4-directions fuzzy division in space
(a) 4 tiles; (b) Membership functions of each tile

Use those four functions, the partition angles between N, E, S, W are $\frac{\pi}{4}$, $\frac{3}{4}\pi$, $\frac{5}{4}\pi$ and $\frac{7}{4}\pi$, and the membership value of those angles is 0.5. The membership value of any angle in interval $\left[0,\frac{\pi}{4}\right]$, $\left[\frac{\pi}{4},\frac{3}{4}\pi\right]$, $\left[\frac{3}{4}\pi,\frac{5}{4}\pi\right]$, $\left[\frac{5}{4}\pi,\frac{7}{4}\pi\right]$, $\left[\frac{7}{4}\pi,2\pi\right]$ can calculated by direction membership functions.

2.2 8-directions fuzzy division

Jin (2009) expressed the transitional part between four main tiles by truth-value gap. The primary direction N contains 60°, and the transitional part contains 30°. It means that each tile in 8-directions model should be unequal, and direction northeast (NE), southeast (SE), southwest (SW) and northwest (NW) should be transitional parts between primary direction N, E, S and W. So we could consider that the 8-directions model is the result of peaking transitional parts. Calculate the intersection of two adjacent directions and peak the intersection, then we can get the membership functions of NE, SE, SW and NW as $\mu_{\mathrm{NE}}(\theta)=2\times\wedge(\mu_{\mathrm{N}},\mu_{\mathrm{E}})$, $\mu_{\mathrm{SE}}(\theta)=2\times\wedge(\mu_{\mathrm{S}},\mu_{\mathrm{E}})$, $\mu_{\mathrm{SW}}(\theta)=2\times\wedge(\mu_{\mathrm{S}},\mu_{\mathrm{W}})$, $\mu_{\mathrm{NW}}(\theta)=2\times\wedge(\mu_{\mathrm{N}},\mu_{\mathrm{W}})$, the membership function of NE can expressed as Eq. (2).

$$\mu_{\mathrm{NE}}(\theta)=\begin{cases}\dfrac{4}{\pi}\theta, & 0\leqslant\theta\leqslant\dfrac{\pi}{4}\\ -\dfrac{4}{\pi}\left(\theta-\dfrac{\pi}{2}\right), & \dfrac{\pi}{4}<\theta\leqslant\dfrac{\pi}{2}\\ 0, & \dfrac{\pi}{2}<\theta\leqslant 2\pi\end{cases}\tag{2}$$

We can get the division angle of N and NE by use their membership functions, and other division angle can be calculated by this method, showed as Table 1. The membership value of each division angle is 0.67, this model divide space like Fig.2.

Table 1 8-directions fuzzy interval division

	N	NE	E	SE	S	SW	W	NW
Interval	$\left[0,\frac{\pi}{6}\right]$, $\left[\frac{11}{6}\pi,2\pi\right]$	$\left[\frac{\pi}{6},\frac{\pi}{3}\right]$	$\left[\frac{\pi}{3},\frac{2}{3}\pi\right]$	$\left[\frac{2}{3}\pi,\frac{5}{6}\pi\right]$	$\left[\frac{5}{6}\pi,\frac{7}{6}\pi\right]$	$\left[\frac{7}{6}\pi,\frac{4}{3}\pi\right]$	$\left[\frac{4}{3}\pi,\frac{5}{3}\pi\right]$	$\left[\frac{5}{3}\pi,\frac{11}{6}\pi\right]$
Angle/(°)	60	30	60	30	60	30	60	30

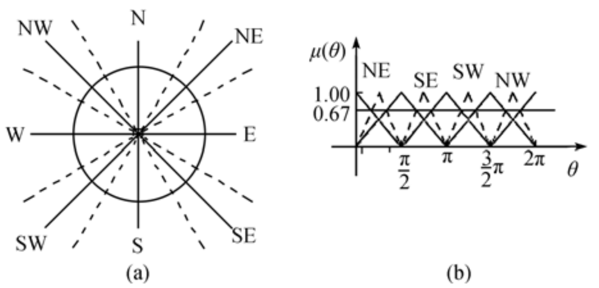

Fig. 2 8-directions fuzzy division in space
(a) 8 tiles; (b) Membership functions of each tile

The method of determining which direction of the angle belongs to is simple. We first calculate the membership value of angle in 8 directions, and then use Eq. (3) to determine the direction of this angle.

$$\mu_{\theta}=\max\left(\mu_A(\theta),\quad \mu_B(\theta),\quad \mu_C(\theta)\right)\tag{3}$$

In Eq. (3), $A,B,C\in\{\mathrm{N,NE,E,SE,S,SW,W,NW}\}$, $A\neq B\neq C$, $\mu_{\theta}\in[0.67\quad 1]$。

This division method is fuzziness base on membership grade, and each direction N, E, S and W has 60° while direction NE, SE, SW and NW has 30° respectively, so it isn't a crisp model. The station of direction N, E, S and W come into prominence, so this model coincident with human cognition. The cognition test about direction concept (Jin *et al*., 2009) has proved this model, and it was called DFAM in this paper.

3 ANALYZE THE UNCERTAINTY OF 8-DIRECTIONS FUZZY ASYMMETRIC MODEL

The membership value of any angle should have error for the inherent positioning error. In this section the uncertainty of

DFAM is analyzed based on interval type-2 fuzzy set theory.

3.1 Calculating the angle deviation

The error of spatial vector data has different attributes since different data sources in GIS. All types of error can be classified into two types: measureable and non-metric error. Support that the error of 2- dimension point follows norm distribution and the positioning error can be expressed by error eclipse in this paper. Use reference point as centre to draw a circle, and the radius of this circle is determined by scale, so this circle is called as basic circle. The deviation of angle θ can be expressed by the distance between two tangents of eclipse, as showed by Fig.3 (a). Those two tangents intersect with the basic circle, and then we can get a short arc. The corresponding angle of this arc is the angle deviation of reference point in direction θ, and θ_N, θ_{NE}, θ_E are the corresponding angle deviation of reference point in directions N, NE, E in Fig.2(a).

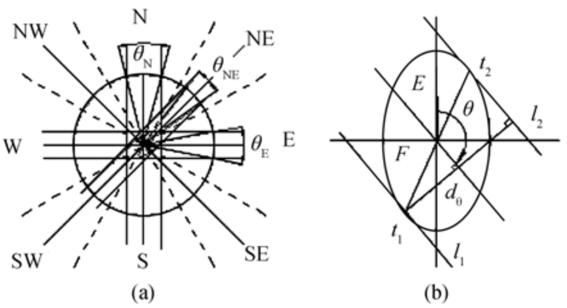

Fig. 3 Direction error of reference point
(a) Calculation of deviation of angle; (b) Distance between two parallel lines

Rotate the eclipse by the reference point and angle $-\theta_0$, as showed by Fig.3 (b), the corresponding angle of θ is $\theta'=\theta-\theta_0$ in Fig.3 (b), and the slope is $K=\tan(\theta')$. So the rotated eclipse can be expressed as:

$$\frac{(x-x_0)^2}{E^2}+\frac{(y-y_0)^2}{F^2}=1 \tag{4}$$

And the slope of any point in this eclipse can be calculated by Eq. (5):

$$K=\frac{F^2}{E^2}\cdot\frac{x-x_0}{y-y_0} \tag{5}$$

Let $K=K'$, two tangent points t_1 and t_2 are calculated by Eq. (4) and Eq. (5):

$$x=\pm\frac{E^2K'}{\sqrt{(EK')^2+F^2}}+x_0 \tag{6}$$

$$y=\pm\frac{F^2}{\sqrt{(EK')^2+F^2}}+y_0 \tag{7}$$

We can get two parallel lines by K', t_1 and t_2, which can be expressed as:

$$\begin{cases} l_1: & y=K'\times(x-x_{t_1})+y_{t_1} \\ l_2: & y=K'\times(x-x_{t_2})+y_{t_2} \end{cases} \tag{8}$$

The distance between l_1 and l_2 is $d_\theta=\frac{\left|K'\times(x_{t_2}-x_{t_1})+y_{t_2}-y_{t_1}\right|}{\sqrt{1+(K')^2}}$. We can get an arc $\overline{t_1t_2}$ by projecting t_1 and t_2 to the base circle, and the length of the arc can be approximately expressed by the central angle.

3.2 Analyze the uncertainty of direction membership function

The positioning error in any direction θ follows the normal distribution, and the variance is $d_\theta/2$, so the angle deviation follows normal distribution and corresponding variance is $\tau_\theta/2$, here $d_\theta\approx\tau_\theta$. Fig.4 (a) shows the probability distribution of

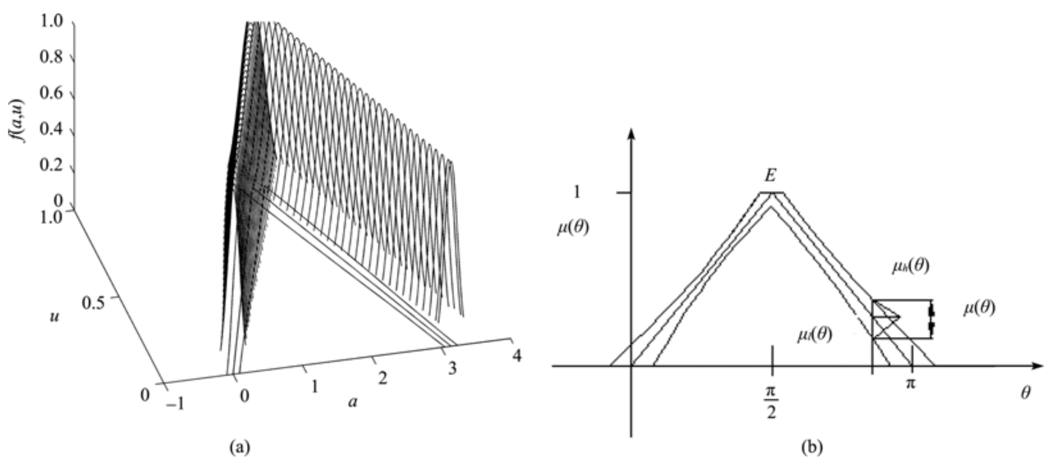

Fig. 4 Type-2 fuzzy set and interval type-2 fuzzy set for east
(a) Type-2 fuzzy set for east; (b) Interval type-2 fuzzy set for east

membership grade of direction E. It has a same membership grade μ_θ for any angle θ in angle interval $[\theta-\tau_\theta/2,\ \theta+\tau_\theta/2]$, but the membership value of μ_θ is a normal function. Map this figure to the horizontal plane, and we can get Fig. 4(b). Fig. 4(b) shows that the membership grade for an angle in fuzzy direction tile E is not a single value but a membership interval $[\mu_l(\theta), \mu_h(\theta)]$, so it means the membership grade has error. Use a vertical plane with x-axis perpendicular to intersect with the surface in Fig. 4(a), then can get a curve. This curve is the secondary membership function.

The type- 2 fuzzy set was introduced by control theory expert L. A. Zadeh for handing uncertainty of membership grade at 1976, and was successfully applied in biology, communication engineering, finance, automation and so on. Mendel and John (2002) defined type- 2 fuzzy set and measurement of uncertainty of it. Type-2 fuzzy set include generalize type-2 fuzzy set and interval type-2 fuzzy set (Chen & Sun, 2005). The curve which is gained in last segment is the secondary membership function. And it means that this fuzzy set is a generalize type-2 fuzzy set apparently. The polygon is the uncertainty zone of it in Fig.4 (b). Fig.6 shows that this model is different from DFAM. It is called extended 8-directions fuzzy asymmetric model (EDFAM) in this paper.

Just as showed by frontal analysis, the secondary membership function of any azimuth angle is a complex curve, and it is difficult to express by an accurate equation. The generalize type-2 fuzzy set is very sophisticated for calculation, and there isn't an acknowledged method for solving this question. There are two simple ways for approximate result: (a) Discretise the primary and secondary membership function at the same time (Coupland & John 2008a, 2008b; Mendel, 2001). (b) Convert the generalize type-2 fuzzy set to interval type-2 fuzzy set, and then discretise the primary membership function (Mendel & Wu, 2005; Greenfield *et al.*, 2009; Coupland & John, 2006). The first way is selected in this paper because we focus on the direction membership error. The footprint of uncertainty (FOU) is used to express the uncertainty of interval type-2 fuzzy set, and it is the union of all primary membership value, so it is the area between upper membership function (UMF) and lower membership function (LMF). The UMF and LMF are type-1 fuzzy membership functions.

The UMF of direction N could be determined by Eq. (9) and Eq. (10), and LMF could be determined by Eq. (11) and Eq. (12). The membership value of UMF equal to 1 while $\frac{\pi}{2}-\frac{\frac{d_\pi}{2}}{2}\leqslant\theta\leqslant\frac{\pi}{2}+\frac{\frac{d_\pi}{2}}{2}$, and this angle interval is called as left and right shoulder. The membership value of LMF is 0 while $-\frac{d_0}{2}\leqslant\theta\leqslant\frac{d_0}{2}$ and $\pi-\frac{d_\pi}{2}\leqslant\theta\leqslant\pi+\frac{d_\pi}{2}$, and called by left or right foot respectively.

$$\mu_{\mathrm{LT}}(\theta)=\frac{2}{\pi}\left(\theta+\frac{d_\theta}{2}\right),-\frac{d_0}{2}\leqslant\theta\leqslant\frac{\pi}{2}-\frac{\frac{d_\pi}{2}}{2} \tag{9}$$

$$\mu_{\mathrm{RT}}(\theta)=-\frac{2}{\pi}\left(\theta-\pi-\frac{d_\theta}{2}\right),\frac{\pi}{2}+\frac{\frac{d_\pi}{2}}{2}\leqslant\theta\leqslant\pi+\frac{d_\pi}{2} \tag{10}$$

$$\mu_{\mathrm{LB}}(\theta)=\frac{2}{\pi}\left(\theta-\frac{d_\theta}{2}\right),\frac{d_0}{2}\leqslant\theta\leqslant\frac{\pi}{2}+\frac{\frac{d_\pi}{2}}{2} \tag{11}$$

$$\mu_{\mathrm{RB}}(\theta)=-\frac{2}{\pi}\left(\theta-\pi+\frac{d_\theta}{2}\right),\frac{\pi}{2}-\frac{\frac{d_\pi}{2}}{2}\leqslant\theta\leqslant\pi-\frac{d_\pi}{2} \tag{12}$$

The uncertainty of membership value of any angle θ belonging to direction N can be calculated by using UMF to minus LMF, expressed as Eq. (13).

$$U=\begin{cases}\mu_{\mathrm{LT}}(\theta), & -\frac{d_0}{2}\leqslant\theta\leqslant\frac{d_0}{2}\\ \mu_{\mathrm{LT}}(\theta)-\mu_{\mathrm{LB}}(\theta), & \frac{d_0}{2}\leqslant\theta\leqslant\frac{\pi}{2}-\frac{\frac{d_\pi}{2}}{2}\\ 1-\mu_{\mathrm{LB}}(\theta), & \frac{\pi}{2}-\frac{\frac{d_\pi}{2}}{2}\leqslant\theta\leqslant\frac{\pi}{2}\\ 1-\mu_{\mathrm{RB}}(\theta), & \frac{\pi}{2}\leqslant\theta\leqslant\frac{\pi}{2}+\frac{\frac{d_\pi}{2}}{2}\\ \mu_{\mathrm{RT}}(\theta)-\mu_{\mathrm{RB}}(\theta), & \frac{\pi}{2}+\frac{\frac{d_\pi}{2}}{2}\leqslant\theta\leqslant\pi-\frac{d_\pi}{2}\\ \mu_{\mathrm{RT}}(\theta), & \pi-\frac{d_\pi}{2}\leqslant\theta\leqslant\pi+\frac{d_\pi}{2}\end{cases} \tag{13}$$

Extend this method to other directions N, NE, E, SE, S, SW, W, NW and we can get the UMF and LMF of each direction tile, then calculate the union of these eight interval type-2 fuzzy sets. The membership and uncertainty is showed as Fig. 6.

3.3 Determine the direction relation between referent point and target

The center of gravity of FOU is calculated directly to determine direction relation if the weight of target in each tiles is not considered. The FOU should be divided into several parts according to atom tiles when the weight is considered, and then some interval type-2 fuzzy sets in each atom tile can be got. The center of each interval type-2 fuzzy set can be calculated by KM algorithm. The result is an interval $\left[c_l(\tilde{A}), c_r(\tilde{A})\right]$, so the center is expressed as:

$$c(\tilde{A})=\frac{c_l(\tilde{A})+c_r(\tilde{A})}{2} \tag{14}$$

The lower membership value of the center of gravity is $\mu_\theta^{\mathrm{LMF}}(c)$, and the upper membership value of it is $\mu_\theta^{\mathrm{UMF}}(c)$.

The uncertainty of EDFAM can be measured by three quanta.

Membership value error:

$$\varepsilon_\alpha(\mu)=\mu^{\mathrm{UMF}}(\alpha)-\mu^{\mathrm{LMF}}(\alpha) \tag{15}$$

Direction center error:

$$\varepsilon(c)=\frac{c_r(\tilde{A})-c_l(\tilde{A})}{2} \tag{16}$$

Membership value error of direction center:

$$\varepsilon_c(\mu)=\mu_\theta^{\mathrm{UMF}}(c)-\mu_\theta^{\mathrm{LMF}}(c) \tag{17}$$

Goyal (2000) proposed the percentage of area of target in each direction could be used as weight to denote the direction relation.

$$p_\theta=\frac{\mathrm{area}(\theta_A\cap B)}{\mathrm{area}(B)} \tag{18}$$

$\theta\in\{\mathrm{N,NE,E,SE,S,SW,W,NW}\}$, "area" represents the area if the target is surface and "area" represents the length while it is line.

When the weight p_θ of target in each tile is considered, then multiply the LMF and UMF of each center by p_θ respectively. A new interval type-2 fuzzy set can be created. Use the KM algorithm to get the center and membership value of this center of the new interval type-2 fuzzy set.

3.4 Compare the EDFAM with cone-based model

The EDFAM model has some similarity with the cone-based model, and the differences between them are as follows:

(1) There are several types of 8-direction cone-based model, but all of them are equal division and each tile has equal angel in these models. EDFAM isn't equal division, and each main direction has about 60 degrees and each secondary direction has about 30 degrees, so this model conform human cognition.

(2) The 4-directions and 8-directions cone-based model haven't hierarchy, but the model proposed by this paper can translate from 8-directions model to 4-directions model easily, and it has hierarchy.

(3) The cone-based model has some properties like transitivity, reflectivity, integrality, relativity and equalization, but EDFAM doesn't have equalization property.

(4) EDFAM considers the positional error of reference point, but the other model doesn't consider it.

(5) EDFAM is more complex than the cone-based model.

4 CASES ANALYSIS

In this section two cases are proposed. The first case is used for analyzing characters of the extended 8-directions fuzzy asymmetric model, and the second one is used for describing the progress of determining direction relation between point and polygon.

4.1 Case 1: characters analysis

There is a line AB and a point P in Fig.5. Let the covariance error matrix are D_1=0, $D_2=\begin{bmatrix}2.91 & 1.25\\1.25 & 1.71\end{bmatrix}\mathrm{m}^2$, $D_3=\begin{bmatrix}3.91 & 0\\0 & 3.91\end{bmatrix}\mathrm{m}^2$, $D_4=\begin{bmatrix}1.73 & 1.24\\1.24 & 4.81\end{bmatrix}\mathrm{m}^2$, $D_5=\begin{bmatrix}8.65 & 3.25\\3.25 & 4.71\end{bmatrix}\mathrm{m}^2$ and the radius of the base circle is 20m, and direction relations between line and reference point are showed in Fig.6(a), (b), (c), (d) and (e) respectively.

Some characters can be induced from Table 2 and Fig.6:

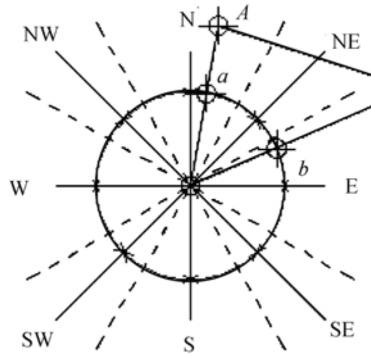

Fig. 5 Fuzzy direction relations between point and line

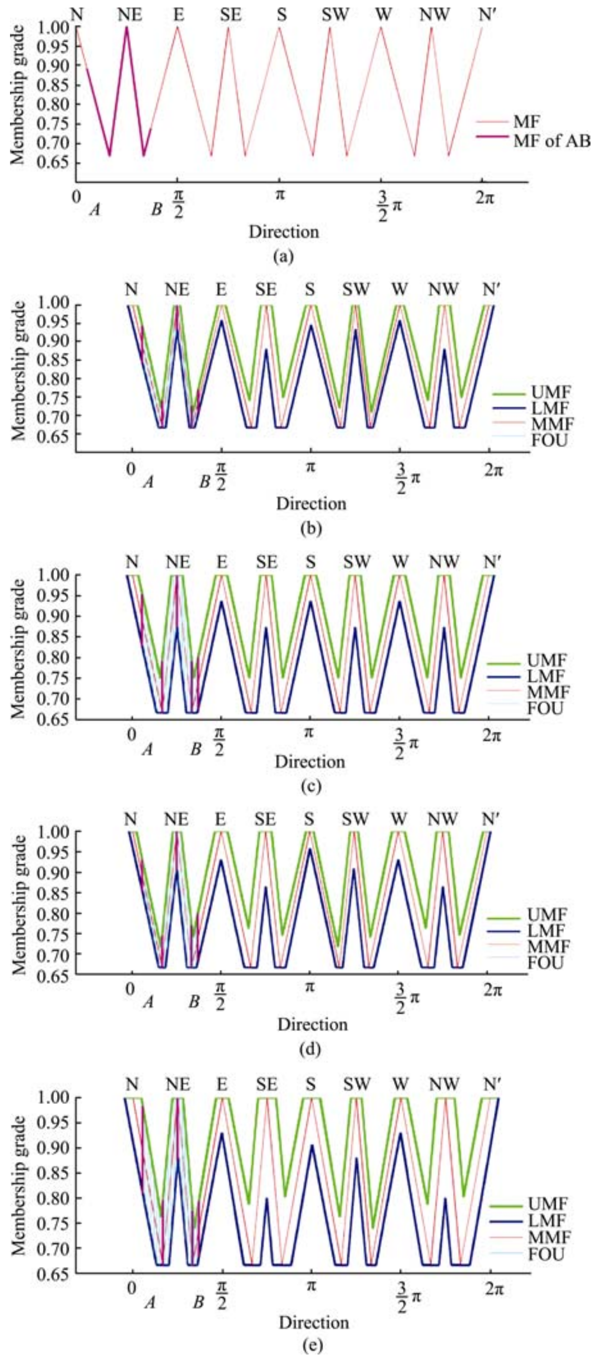

Fig. 6 Comparison of direction membership error

Table 2 Comparison analysis of line's direction membership error

Positioning error	Angle	0.17	0.52	0.79	1.05	1.16
	Direction	N	Division angle	NE	Division angle	E
D_1	min	0.8919	0.6670	1.0000	0.6670	0.7387
	max	0.8919	0.6670	1.0000	0.6670	0.7387
	error	0	0	0	0	0
D_2	min	0.8440	0.6668	0.9323	0.6668	0.7074
	max	0.9439	0.7419	1.0000	0.7279	0.7709
	error	0.0999	0.0751	0.0677	0.0610	0.0634
D_3	min	0.8288	0.6668	0.8741	0.6668	0.6755
	max	0.9547	0.7926	1.0000	0.7926	0.8014
	error	0.1259	0.1257	0.1259	0.1257	0.1259
D_4	min	0.8549	0.6668	0.9040	0.6668	0.6828
	max	0.9307	0.7454	1.0000	0.7704	0.8004
	error	0.0758	0.0785	0.0960	0.1035	0.1177
D_5	min	0.8107	0.6668	0.8746	0.6668	0.6844
	max	0.9845	0.7973	1.0000	0.7760	0.7950
	error	0.1738	0.1305	0.1254	0.1092	0.1106

(1) EDFAM is not equal division for space. Each main direction N, E, S and W has 60° and each secondary direction NE, SE, SW, and NW has 30°.

(2) The membership grade error of secondary directions is larger than that of main directions, so it means that the secondary directions are vaguer than main directions. This result is consistent with the result of method based on fuzzy entropy.

(3) The positioning error of reference point increase the uncertainty of direction relation, but this increased error is objectively exists. It means that the extended 8-directions fuzzy asymmetric model can objectively describe fuzzy direction relation.

(4) The direction membership grade error will increase if the positioning error of reference point increase, but the uncertainty of secondary directions increases seriously than that of main directions.

(5) The direction membership grade error of any angle is symmetry while $\sigma_x \neq \sigma_y$, namely $\varepsilon_\alpha(\mu)=\varepsilon_{\alpha+\pi}(\mu)$, but $\varepsilon_\alpha(\mu)=\varepsilon_\alpha(\mu)=\varepsilon_{\alpha+\frac{\pi}{2}}(\mu)=\varepsilon_{\alpha+\pi}(\mu)=\varepsilon_{\alpha+\frac{3\pi}{2}}(\mu)$ while $\sigma_x=\sigma_y$. The membership grade error of main directions is smaller than that of secondary directions, because the secondary directions are transitional partition of primary directions, so they should fuzzier than primary directions. The membership grade error of each direction equal to 0 when $\sigma_x=\sigma_y=0$.

In this table min means minimal membership value, max means maximal membership value, and error means membership value error.

4.2 Case 2: determine the direction relation between reference point with positioning error and polygon

The covariance error matrix of reference point A is $D_A=\begin{bmatrix}10.91 & 3.25\\ 3.25 & 4.71\end{bmatrix}\text{m}^2$ and the radius of base circle is 20m in Fig.7, and polygon B is the target object. The process of calculating the direction relation between A and B is showed in Table 3. The direction relation between A and B can be determined after getting the direction center CS based on Table 1, and the uncertainty of the direction relation can be described by Eq. (15)—Eq. (17).

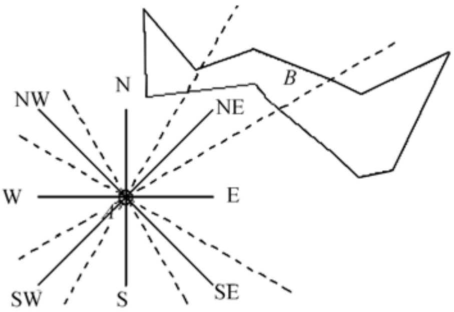

Fig. 7 Direction relationship between reference point with position error and complex objects

Table 3 Direction relationship reasoning between reference point with position error and polygon

	S_1	S_2	S_3
Direction	N	NE	E
Start angle	0.1047	0.5236	1.0472
End angle	0.5236	1.0472	1.6581
Area	12718.087	19269.955	45874.891
Area weight p_l	0.1633	0.2475	0.5892
c_l	0.2914	0.7631	1.3522
c_r	0.3076	0.8042	1.3824
c	0.2995	0.7837	1.3673
$\mu_\theta(c)$	0.8052	0.9049	0.8732
$\mu_\theta^{\text{LMF}}(c)$	0.7400	0.8098	0.7643
$\mu_\theta^{\text{UMF}}(c)$	0.8704	1.0000	0.9821
$p_\theta \times \mu_\theta(c)$	0.1315	0.2240	0.5145
$p_\theta \times \mu_\theta^{\text{LMF}}(c)$	0.1209	0.2004	0.4503
$p_\theta \times \mu_\theta^{\text{UMF}}(c)$	0.1422	0.2475	0.5786
CL		1.0146	
CS		1.0543	
CR		1.0939	

5 CONCLUSION

Qualitative direction relation representation and reasoning is complex because of the vagueness, hierarchy of concept of direction and uncertainty of spatial data, but existing models use sample methods to describe and reason direction relation. The 8-directions fuzzy asymmetric model (DFAM) is introduced at first. The positioning error of reference point should be considered in direction relation description and reasoning because this type of error affects the result directly. The membership grade error is calculated based on positioning error of reference point. The extended 8-directions fuzzy asymmetric model (EDFAM) is developed which takes the positioning error of reference point and vagueness of direction concept based on interval type-2 fuzzy set into account, and the method to analyze the direction relation between point and polygon is developed too.

The methods to model fuzzy object usually contain some subjective factors under certain degree in classical fuzzy set applications. The membership grade error can be described by type-2 fuzzy set, so it is more objective than classical fuzzy set. The generalize type-2 fuzzy set is computational complexity, but the interval type-2 fuzzy set can reduce the computational complexity. So the interval type-2 fuzzy set is more suitable for modeling fuzzy object or describing direction relation.

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八方向模糊不均匀划分及参考点位误差所致不确定性分析

郭继发[1,2], 崔伟宏[1]

1. 中国科学院 研究生院, 北京, 100491
2. 中国科学院 遥感应用研究所,北京, 100101;

摘 要: 方向概念的模糊性和空间数据固有的不确定性导致了方向关系的复杂性, 在空间关系描述和推理研究中需要考虑空间数据的不确定性和方向概念的模糊性。在四方向模型中各方向片是等角划分; 在八方向模型中 4 个主要方向片各占 60°, 4 个次要方向片各占 30°。利用区间二型模糊集理论建立了顾及参考点点位误差的八方向模糊不均匀划分模型, 基于区间二型模糊集讨论了方向主隶属度成员函数和隶属度的不确定性。对比分析了八方向模糊不均匀划分模型与锥形模型的区别, 讨论了具有点位误差的参考点与线和多边形的方向关系计算过程, 通过两个实例分析了该模型的特点和点与多边形方向关系的确定方法。

关键词: 方向关系, 区间二型模糊集, 方向隶属度误差

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1 引 言

方向关系的近似描述和推理是地理信息科学和人工智能等领域重要的一个研究方向, 许多学者在这方面做了大量的研究, 取得了丰硕的成果。目前的方向关系模型包括锥形模型、最小外接矩形模型、2-D String 模型、Freksa-Zimmermann 模型、方向关系矩阵模型和 Voronio 模型(郭庆胜等, 2006; 杜世宏等, 2007)。一些研究人员运用模糊集理论、粗糙集理论和概率论进行了方向关系的近似描述研究(Papadias 等, 1999; 杜世宏等, 2004, 2005; 曹菡等, 2001; Goyal, 2000; 杜世宏 & 王桥, 2004; Liu, 2008), 取得了可喜的成果。部分学者从认知的角度, 用实验的方法考察了人们对方向概念的模糊性(Montello & Frank, 1996; 金鑫等, 2009)。

由于方向关系的近似描述和推理受到多方面的制约, 笔者认为目前在空间方向关系描述中存在以下问题:

(1) 现有的模型不能体现方向概念的模糊性, 与人们的认知习惯不符, 金鑫等(2009)的实验结果说明了这一点。

(2) 现有的模型如锥形模型、MBR 模型均是对空间的硬性划分, 各方向片之间没有过渡; 一些文献采用的以“一定宽度的区间”作为主方向间的过渡带或作为模糊区间, 而这一做法带有主观性。

(3) 对于方向片内部目标实体的方向隶属程度不能很好地表达。

(4) 没有为各原子方向建立一套有效的模糊隶属函数, 由于空间数据不可避免的存在位置误差, 由此建立的原子方向隶属度也有误差, 而且方向误差还与参考对象和目标对象之间的距离有关, 但目前的研究没有对此进行讨论。

(5) 地理实体方向关系的表达跟自然语言描述不符。

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第一作者简介: 郭继发(1981—), 男, 博士, 讲师, 2010 年毕业于中国科学院研究生院, 获理学博士学位, 目前主要从事模糊地理信息建模和分析方面的研究工作, 发表论文 10 余篇。E-mail: guojfx2004@163.com。

2　不考虑参考点点位误差的方向模糊划分

在现有的研究成果中，四方向划分的锥形模型和八方向划分的锥形模型是彼此独立的。根据人们的认知习惯，八方向锥形模型应该是四方向锥形模型的细化，在此基础上还可以进行十六或三十二方向的划分。

2.1　空间四方向模糊划分

在空间二维直角坐标系中，空间四方向模糊划分如图 1(a)(郭庆胜　等, 2006)。各方向的隶属函数如图 1(b)，其中方向 N 的隶属函数为公式(1)，类似的可以建立 E, S, W 三方向的隶属函数。通过联立相邻方向的隶属函数，可解得 E, S, W, N 四方向的模糊划分角度分别为 $\frac{\pi}{4}$，$\frac{3}{4}\pi$，$\frac{5}{4}\pi$，$\frac{7}{4}\pi$；分界处的隶属度均为 0.5。任意方位角在区间[0, $\frac{\pi}{4}$]；[$\frac{\pi}{4}$，$\frac{3}{4}\pi$]；[$\frac{3}{4}\pi$，$\frac{5}{4}\pi$]；[$\frac{5}{4}\pi$，$\frac{7}{4}\pi$]；[$\frac{7}{4}\pi$，2π]的隶属度可通过方向隶属函数求得。

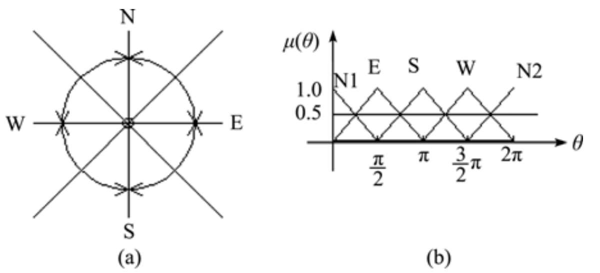

图 1　空间四元方向模糊划分

(a) 四方向划分; (b) 四方向模糊隶属函数

$$\mu_N(\theta)=\begin{cases}-\dfrac{2}{\pi}\left(\theta-\dfrac{\pi}{2}\right), & 0\leqslant\theta\leqslant\dfrac{\pi}{2}\\ 0, & \dfrac{\pi}{2}<\theta\leqslant\dfrac{3}{2}\pi\\ \dfrac{2}{\pi}\left(\theta-\dfrac{3}{2}\pi\right), & \dfrac{3}{2}\pi<\theta\leqslant 2\pi\end{cases}\tag{1}$$

2.2　空间八方向模糊划分

金鑫等(2009)中用“真值间隙”理论描述了主方向片之间的过渡区间，主方向“N”的方向角大约为 60°，而过渡区间大约为 30°，这说明了在八方向划分中各方向片不应该是等角划分。而 NE, SE, SW, NW 4 个方向正是 N, E, S, W 4 个主方向的过渡部分。因此可以认为八方向划分是突出图 1(a)中隶属度为 0.5 附近的角度区间。先求相邻方向的交集，并利用模糊集理论中的锐化方法，取 $\mu_{\mathrm{NE}}(\theta)=2\times\wedge(\mu_{\mathrm{N}},\mu_{\mathrm{E}})$，$\mu_{\mathrm{SE}}(\theta)=2\times\wedge(\mu_{\mathrm{S}},\mu_{\mathrm{E}})$，$\mu_{\mathrm{SW}}(\theta)=2\times\wedge(\mu_{\mathrm{S}},\mu_{\mathrm{W}})$，$\mu_{\mathrm{NW}}(\theta)=2\times\wedge(\mu_{\mathrm{N}},\mu_{\mathrm{W}})$，得到 NE, SE, SW, NW 4 个方向的隶属函数，其中 NE 方向的隶属函数可用公式(2)描述。

$$\mu_{\mathrm{NE}}(\theta)=\begin{cases}\dfrac{4}{\pi}\theta, & 0\leqslant\theta\leqslant\dfrac{\pi}{4}\\ -\dfrac{4}{\pi}\left(\theta-\dfrac{\pi}{2}\right), & \dfrac{\pi}{4}<\theta\leqslant\dfrac{\pi}{2}\\ 0, & \dfrac{\pi}{2}<\theta\leqslant 2\pi\end{cases}\tag{2}$$

通过联立 N 和 NE 方向隶属函数，可求得 N 和 NE 方向的模糊划分，类似的可求八元方向的模糊划分，如表 1，在各分界点的隶属度均为 0.67，八方向对空间的划分如图 2。

表 1　八方向模糊区间划分

	N	NE	E	SE	S	SW	W	NW
区间	$[0,\frac{\pi}{6}],[\frac{11}{6}\pi,2\pi]$	$[\frac{\pi}{6},\frac{\pi}{3}]$	$[\frac{\pi}{3},\frac{2}{3}\pi]$	$[\frac{2}{3}\pi,\frac{5}{6}\pi]$	$[\frac{5}{6}\pi,\frac{7}{6}\pi]$	$[\frac{7}{6}\pi,\frac{4}{3}\pi]$	$[\frac{4}{3}\pi,\frac{5}{3}\pi]$	$[\frac{5}{3}\pi,\frac{11}{6}\pi]$
夹角/(°)	60	30	60	30	60	30	60	30

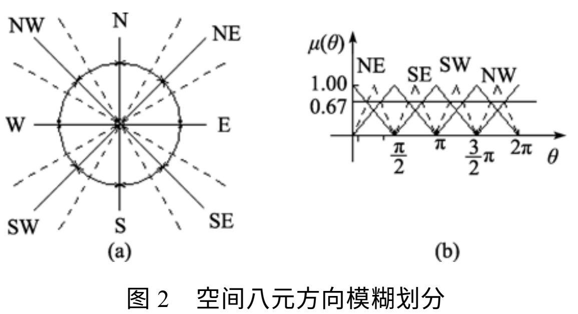

图 2　空间八元方向模糊划分

(a) 八方向划分; (b) 八方向模糊隶属函数

要确定某方位角属于某方向，首先用各原子方向的隶属函数求得任意方位角在八区间的隶属度，然后利用式(3)确定属于某方向。

$$\mu_\theta=\max(\mu_A(\theta),\mu_B(\theta),\mu_C(\theta))\tag{3}$$

式(3)中，$A,B,C\in\{\mathrm{N,NE,E,SE,S,SW,W,NW}\}$　$A\neq B\neq C$，$\mu_\theta\in[0.67\ 1]$。

这种划分是基于隶属度的模糊划分，其中 N, E, S, W 各占 60°，而 NE, SE, SW, NW 各占 30°，所以该

模型不是空间方向的硬性划分，突出了 E, S, W, N 四方向的主要地位，符合人们的认知习惯，金鑫等(2009)的认知实验验证了本文的方向划分方法和各方向片隶属函数的正确性。

3 参考点具有点位误差时八方向模糊划分的不确定性分析

参考点不可避免的存在点位误差，在参考点的各方向均存在一定的偏差。

3.1 任意方向角偏差的计算

在 GIS 的矢量数据中，由于数据来源不同，误差具有不同的特征，包括可度量和不可度量的误差。假定二维随机点点位误差服从正态分布，位置不确定性用误差椭圆表示。以参考点为圆心，以一定的半径建立参考圆，参考圆的半径由地图的尺度确定。参考点在任意方向θ上的位置偏差可用误差椭圆在θ方向上两条切线之间的范围表示，如图 3(a)。为了求得方向偏差，两条方向切线跟参考圆相交，得到一段圆弧，该圆弧对应的夹角即为方向偏差，图 3(a)中的θ_N, θ_{NE}, θ_E 就是参考点在 N, NE, E 3 个方向上的方向偏差。

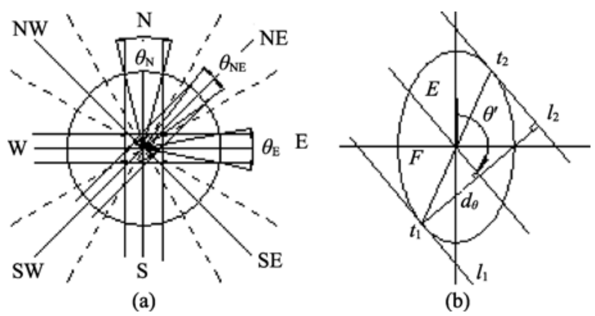

图 3 参考点的方向误差

(a) 方向偏差的计算; (b) 方向切线间的距离

以(x_0, y_0)为圆心、椭圆倾斜角的负值$(-\theta_0)$旋转误差椭圆，得到标准误差椭圆，如图 3(b)。相应的任意方向角为$\theta'=\theta-\theta_0$，该方向上切线斜率为 $K=\tan(\theta')$。该点对应的误差椭圆方程为:

$$\frac{(x-x_0)^2}{E^2}+\frac{(y-y_0)^2}{F^2}=1 \tag{4}$$

该误差椭圆上任意点的斜率为:

$$K=\frac{F^2}{E^2}\frac{x-x_0}{y-y_0} \tag{5}$$

取 $K=K'$，联立方程(4)、方程(5)，即可求得椭圆上两个斜率为 K 的点:

$$x=\pm\frac{E^2K'}{\sqrt{(EK')^2+F^2}}+x_0 \tag{6}$$

$$y=\pm\frac{F^2}{\sqrt{(EK')^2+F^2}}+y_0 \tag{7}$$

求得两点 t_1, t_2；由 K'和 t_1, t_2 得到一组平行线 l_1, l_2:

$$\begin{cases} l_1: \quad y=K'\times\left(x-x_{t_1}\right)+y_{t_1} \\ l_2: \quad y=K'\times\left(x-x_{t_2}\right)+y_{t_2} \end{cases} \tag{8}$$

两平行线的距离为: $d_\theta=\dfrac{\left|K'\times\left(x_{t_2}-x_{t_1}\right)+y_{t_2}-y_{t_1}\right|}{\sqrt{1+\left(K'\right)^2}}$。

将点 t_1, t_2 投影到单位圆；可以得到圆弧$\overline{t_1t_2}$，那么$\overline{t_1t_2}$ 的弧长可近似表示$\overline{t_1t_2}$ 的圆心角。即 d_θ是参考点在任意方向θ上的方向偏差τ_θ。

3.2 方向隶属函数的不确定性分析

参考点的点位误差在任意方向轴上均服从正态分布，且方差为 $d_\theta/2$。投影到参考圆上后得到的方向偏差也服从正态分布，且方差为$\tau_\theta/2$，图 4(a)为模糊方向 E 的隶属度的概率分布。那么对于任意方位角θ，在区间$[\theta-\tau_\theta/2, \theta+\tau_\theta/2]$，均有相同的隶属度$\mu_\theta$。在图 4(a)中将该区间投影到水平面上，即可得到如图 4(b)所示的图形。从图 4(b)看出，任意方位角对模糊方向 E 的隶属度$\mu(\theta)$是一个隶属区间$[\mu_l(\theta), \mu_h(\theta)]$，说明隶属度存在误差。

模糊二型理论由美国控制论专家 Zadeh 针对数据的不确定性在 1976 年提出，已成功应用于生物、通信、金融和自动控制等领域, Mendel 和 John(2002)给出了二型模糊集的定义和主隶属度不确定性的度量方法。二型模糊集可分为一般二型模糊集和区间二型模糊集，其中一般二型模糊集是指二型模糊集中次隶属成员函数为一般函数的模糊集；当次隶属函数是一型区间模糊集时成为区间二型模糊集(陈薇 & 孙增圻, 2005)。在图 5(a)中以垂直 x 轴作一竖直面与曲面相交，其交线是一任意曲线，该曲线即为二型模糊集的次隶属函数，显然，该方向模糊集是一般二型模糊集，图 5(b)中多边形所围的区域即为该二型模糊集的主隶属函数的不确定性区域。

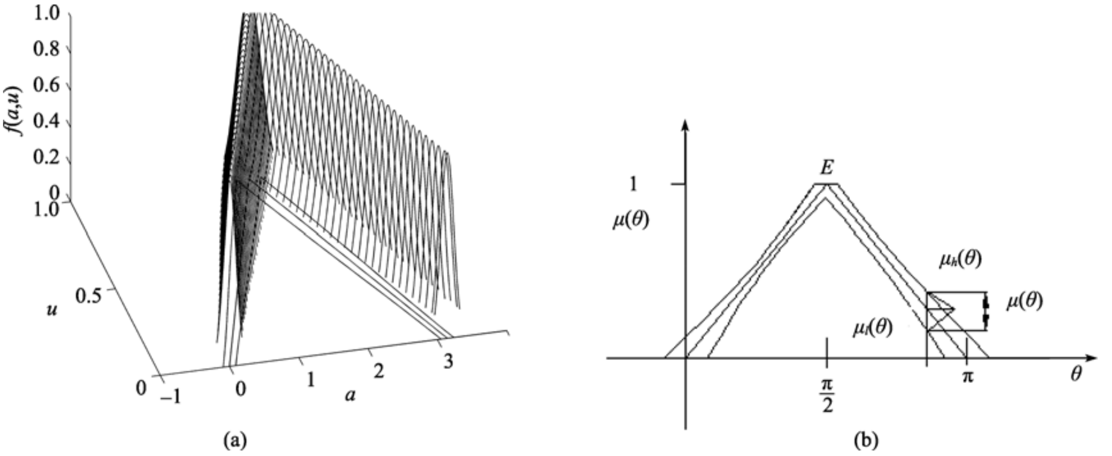

图 4　东方向二型模糊集

(a) 表示方向东的二型模糊集; (b) 表示方向东的区间二型模糊集

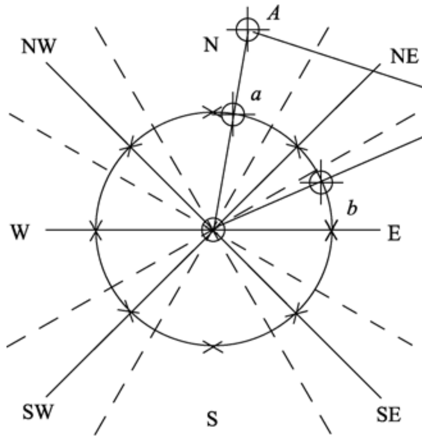

图 5　点线模糊方向关系

从上面分析看出，任意方位角的次隶属函数是一复杂曲线，难以用数学公式表示，在实际应用中，需对其简化。目前对一般二型模糊集的求解是其研究的一个难点，尚未有一种公认的合理方法，对于一般二型模糊集的简化通常有两种方法：(1) 将一般二型模糊集的主隶属函数和次隶属函数的离散化(Coupland & John 2008a, 2008b; Mendel, 2001)。(2) 将一般二型模糊集简化为二型区间模糊集，然后对主隶属函数离散化(Mendel & Wu, 2005; Greenfield 等, 2009; Coupland & John, 2006)。在此关心的是方向隶属度的误差，因此，我们可以采用第一种方法，将主隶属度在区间[$\mu_l(\theta)$, $\mu_h(\theta)$]内的次隶属度值设为 1，区间外的值设为 0，这样把该一般二型模糊集转化为区间二型模糊集。在区间二型模糊集中以“足迹”表示区间二型模糊集的不确定性区域(Mendel & Wu, 2005)，称为不确定性的足迹(footprint of uncertainty, FOU)，它是所有主隶属度值的并集，即为上隶属函数(UMF)和下隶属函数(LMF)之间的区域，其中为 UMF 和 LMF 均为一型模糊集。

本文中 UMF 通过式(9)、式(10)确定。LMF 通过式 (11)、式(12)确定。当 $\frac{\pi}{2}-\frac{d_{\frac{\pi}{2}}}{2}\leqslant\theta\leqslant\frac{\pi}{2}+\frac{d_{\frac{\pi}{2}}}{2}$ 时隶属度为 1，称为左、右肩。当 $-\frac{d_0}{2}\leqslant\theta\leqslant\frac{d_0}{2}$，$\pi-\frac{d_\pi}{2}\leqslant\theta\leqslant\pi+\frac{d_\pi}{2}$ 时隶属度为 0，称为左、右底。

$$\mu_{\mathrm{LT}}(\theta)=\frac{2}{\pi}\left(\theta+\frac{d_\theta}{2}\right),\ -\frac{d_0}{2}\leqslant\theta\leqslant\frac{\pi}{2}-\frac{d_{\frac{\pi}{2}}}{2} \tag{9}$$

$$\mu_{\mathrm{RT}}(\theta)=-\frac{2}{\pi}\left(\theta-\pi-\frac{d_\theta}{2}\right),\ \frac{\pi}{2}+\frac{d_{\frac{\pi}{2}}}{2}\leqslant\theta\leqslant\pi+\frac{d_\pi}{2} \tag{10}$$

$$\mu_{\mathrm{LB}}(\theta)=\frac{2}{\pi}\left(\theta-\frac{d_\theta}{2}\right),\ \frac{d_0}{2}\leqslant\theta\leqslant\frac{\pi}{2}+\frac{d_{\frac{\pi}{2}}}{2} \tag{11}$$

$$\mu_{\mathrm{RB}}(\theta)=-\frac{2}{\pi}\left(\theta-\pi+\frac{d_\theta}{2}\right),\ \frac{\pi}{2}-\frac{d_{\frac{\pi}{2}}}{2}\leqslant\theta\leqslant\pi-\frac{d_\pi}{2} \tag{12}$$

对于任意角θ属于 N 的隶属度的不确定性可用上边界跟下边界的差值来度量，可用公式(13)计算。

$$U=\begin{cases}\mu_{\mathrm{LT}}(\theta), & -\frac{d_0}{2}\leqslant\theta\leqslant\frac{d_0}{2}\\ \mu_{\mathrm{LT}}(\theta)-\mu_{\mathrm{LB}}(\theta), & \frac{d_0}{2}\leqslant\theta\leqslant\frac{\pi}{2}-\frac{d_{\frac{\pi}{2}}}{2}\\ 1-\mu_{\mathrm{LB}}(\theta), & \frac{\pi}{2}-\frac{d_{\frac{\pi}{2}}}{2}\leqslant\theta\leqslant\frac{\pi}{2}\\ 1-\mu_{\mathrm{RB}}(\theta), & \frac{\pi}{2}\leqslant\theta\leqslant\frac{\pi}{2}+\frac{d_{\frac{\pi}{2}}}{2}\\ \mu_{\mathrm{RT}}(\theta)-\mu_{\mathrm{RB}}(\theta), & \frac{\pi}{2}+\frac{d_{\frac{\pi}{2}}}{2}\leqslant\theta\leqslant\pi-\frac{d_\pi}{2}\\ \mu_{\mathrm{RT}}(\theta), & \pi-\frac{d_\pi}{2}\leqslant\theta\leqslant\pi+\frac{d_\pi}{2}\end{cases} \tag{13}$$

将该方法扩展到 N, NE, E, SE, S, SW, W, NW 等八个模糊方向，然后求各其并集，并以 0.67 做截集。得到各方向的模糊隶属度分布如图 6，图 6 中 LMF 为上隶属函数，UMF 为下隶属函数，通过两条曲线可确定参考点在任意方向上的方向隶属度误差。

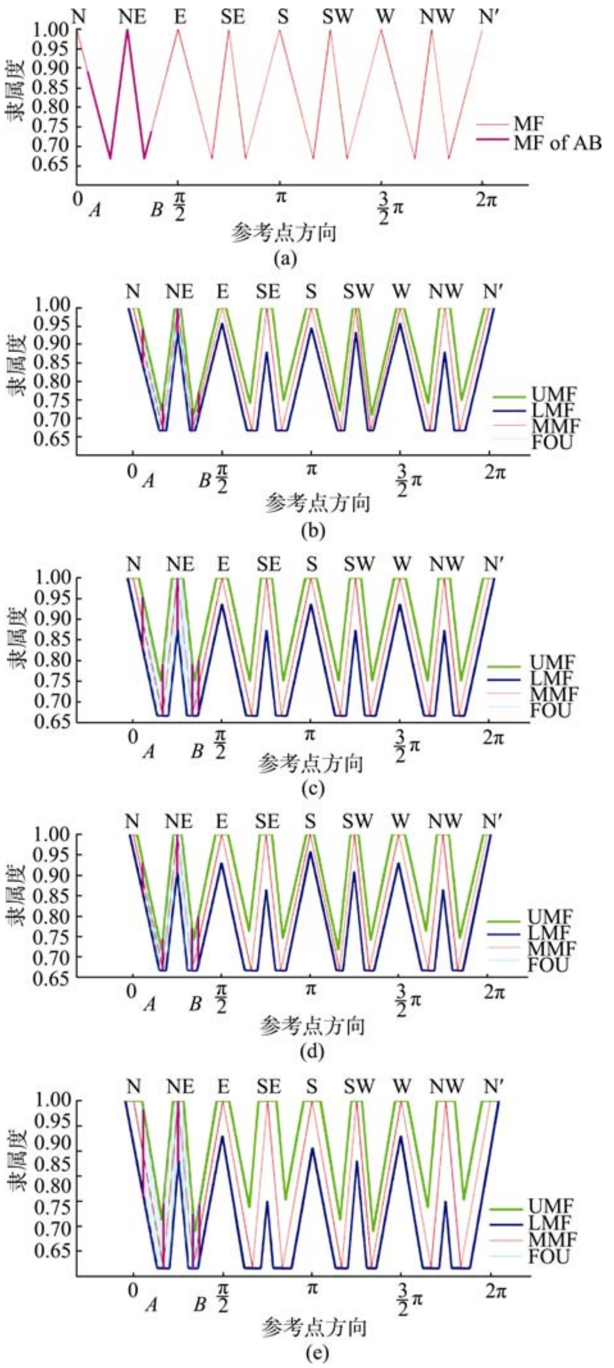

图 6 方向隶属度误差对比

3.3 点与目标对象的方向关系确定方法

如果不顾及目标对象在原子方向片中的权重，可以直接通过求 FOU 的重心确定其方向关系。在顾及权重的情况下，首先将不确定区间划分按各原子区间划分为若干部分，得到原子区间上的区间二型模糊集，然后采用 KM 降型算法逐个对原子区间二型模糊集解模糊求其中心，对于二型模糊集降型后的中心为区间$\left[c_l\left(\tilde{A}\right),c_r\left(\tilde{A}\right)\right]$，最后的方位中心为:

$$c\left(\tilde{A}\right)=\frac{c_l\left(\tilde{A}\right)+c_r\left(\tilde{A}\right)}{2} \tag{14}$$

进一步求得各方向中心 c 的 LMF 值为 $\mu_\theta^{\mathrm{LMF}}(c)$，UMF 值为 $\mu_\theta^{\mathrm{UMF}}(c)$。

模型的不确定性可由以下 3 个量进行度量，

任意方向的隶属度误差

$$\varepsilon_\alpha(\mu)=\mu^{\mathrm{UMF}}(\alpha)-\mu^{\mathrm{LMF}}(\alpha) \tag{15}$$

方位中心误差

$$\varepsilon\left(c\right)=\frac{c_r\left(\tilde{A}\right)-c_l\left(\tilde{A}\right)}{2} \tag{16}$$

方位中心的隶属度误差

$$\varepsilon_c(\mu)=\mu_\theta^{\mathrm{UMF}}(c)-\mu_\theta^{\mathrm{LMF}}(c) \tag{17}$$

完成计算各原子方向的中心后，需要确定点与复杂对象的概略方向关系。Goyal(2000)提出以目标对象落在参照对象的各个方向区域内的面积与目标对象总面积的百分比来表示方向关系。在此，利用这一百分比为权重确定推理参考点与复杂对象的方向关系。

$$p_\theta=\frac{\mathrm{area}\left(\theta_A\cap B\right)}{\mathrm{area}\left(B\right)} \tag{18}$$

式中，$\theta\in\{\mathrm{N,NE,E,SE,S,SW,W,NW}\}$，当目标对象为面状对象时，area 表示计算面积；为线状对象时，area 表示计算长度。

为了顾及目标对象在各方向片中的权重，将 LMF、UMF 对应的乘 p_θ，利用然后再一次利用 KM 降型法确定整个 FOU 的中心。

3.4 与锥形模型的对比分析

本文提出的模型跟锥形模型存在相似的地方，在此讨论跟锥形模型的区别。

(1) 八方向划分的锥形模型有多种，但都是等区间划分；而本文提出的模型为不均匀划分，该方法更符合人们的认知习惯。

(2) 锥形模型的四方向划分和八方向划分没有层次性；本文提出的八方向模糊不均匀划分模型可以很容易的向四方向划分模型转化，具有层次性。

(3) 锥形模型具有传递性、反射性、完整性、相对性、平等性等特征；八方向模糊不均匀划分模型不具备平等性特征，但具有其他特征。

(4) 锥形模型没有考虑参考点点位误差；而本文的研究顾及了参考点点位误差。

(5) 顾及了参考点点位误差的八方向模糊不均匀划分模型计算量比锥形模型大。

4 实例分析

4.1 实例 1

如图 5 所示，考虑参考点点位误差，参考圆半径为 20m，当参考点协方差阵分别为 $D_1=0$，$D_2=\begin{bmatrix}2.91 & 1.25\\1.25 & 1.71\end{bmatrix}$ m^2，$D_3=\begin{bmatrix}3.91 & 0\\0 & 3.91\end{bmatrix}$ m^2，$D_4=\begin{bmatrix}1.73 & 1.24\\1.24 & 4.81\end{bmatrix}\mathrm{m}^2$，$D_5=\begin{bmatrix}8.65 & 3.25\\3.25 & 4.71\end{bmatrix}\mathrm{m}^2$ 时；直线和参考点的方向关系分别如图 6(a), (b), (c), (d), (e)所示。

在图 6 中，红色曲线表示不考虑参考点点位误差时的隶属度分布函数，为一型模糊函数；LMF 表示方向关系隶属度上界，UMF 表示方向关系隶属度上界，由上界和下界围成的区域(表示方向关系隶属度上界)描述了参考点的方向隶属度误差分布。直线 AB 上 5 个点的方向隶属度及误差如表 2，整个填充区域为参考点与 AB 的方向关系不确定性区域。

表 2 直线的方向隶属度误差对比分析

参考点误差	角度	0.17	0.52	0.79	1.05	1.16
	隶属方向	N	分界点	NE	分界点	E
D_1	隶属度下界	0.8919	0.6670	1.0000	0.6670	0.7387
	隶属度上界	0.8919	0.6670	1.0000	0.6670	0.7387
	隶属度误差	0	0	0	0	0
D_2	隶属度下界	0.8440	0.6668	0.9323	0.6668	0.7074
	隶属度上界	0.9439	0.7419	1.0000	0.7279	0.7709
	隶属度误差	0.0999	0.0751	0.0677	0.0610	0.0634
D_3	隶属度下界	0.8288	0.6668	0.8741	0.6668	0.6755
	隶属度上界	0.9547	0.7926	1.0000	0.7926	0.8014
	隶属度误差	0.1259	0.1257	0.1259	0.1257	0.1259
D_4	隶属度下界	0.8549	0.6668	0.9040	0.6668	0.6828
	隶属度上界	0.9307	0.7454	1.0000	0.7704	0.8004
	隶属度误差	0.0758	0.0785	0.0960	0.1035	0.1177
D_5	隶属度下界	0.8107	0.6668	0.8746	0.6668	0.6844
	隶属度上界	0.9845	0.7973	1.0000	0.7760	0.7950
	隶属度误差	0.1738	0.1305	0.1254	0.1092	0.1106

从上面的分析可以看出:

(1) 基于隶属度的八元模糊方向划分不是等角划分，其中 N, E, S, W 4 个主要方向各占 60°，而 NE, SE, SW, NW 次要方向各占 30°，符合人们习惯。

(2) 当考虑参考点点位误差时，次方向的隶属度误差比主方向的隶属度误差大，说明作为主方向过渡方向的次方向有比主方向更大的模糊性，这一分析结果跟基于模糊熵的分析结果是一致的。

(3) 由于参考点点位误差引起方向隶属度误差，增加了方向关系的不确定性，但这种增加的不确定性客观存在，说明模糊方向的二型模糊集描述模型能客观的描述模糊方向关系。

(4) 当参考点误差增大时方向隶属度误差增大，但是次方向隶属度误差增大更明显。

(5) 当 $\sigma_x \neq \sigma_y$ 时，任意方向的隶属度误差表现为对称性，即 $\varepsilon_\alpha(\mu)=\varepsilon_{\alpha+\pi}(\mu)$；当 $\sigma_x=\sigma_y$ 时，$\varepsilon_\alpha(\mu)=\varepsilon_\alpha(\mu)=\varepsilon_{\alpha+\frac{\pi}{2}}(\mu)=\varepsilon_{\alpha+\pi}(\mu)=\varepsilon_{\alpha+\frac{3\pi}{2}}(\mu)$，但是主方向上的隶属度误差小于等于次方向上的隶属度误差，因为次方向作为主方向的过度区间，具有较大的模糊性，特别的当 $\sigma_x=\sigma_y=0$ 时，主方向和次方向上的隶属度误差均等于 0。

4.2 顾及参考点点位误差时点与多边形方向关系的确定

如图 7 所示，参考点 A 的误差阵为 $D_A=\begin{bmatrix}10.91 & 3.25\\3.25 & 4.71\end{bmatrix}\mathrm{m}^2$，参考圆半径为 20m, B 为多边形对象。B 落入参考点 A 的 N, NE, E 3 个原子方向中。在考虑参考点点位误差时 A 与 B 的方向关系计算过程如表 3。

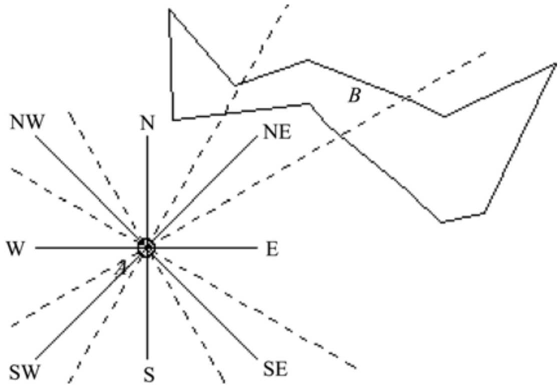

图 7 带有误差的参考点与复杂对象的方向关系

表 3 带有误差的参考点与复杂线的方向关系推理

	S_1	S_2	S_3
隶属方向	N	NE	E
起始角	0.1047	0.5236	1.0472
终止角	0.5236	1.0472	1.6581
面积	12718.087	19269.955	45874.891
面积权重 p_l	0.1633	0.2475	0.5892
c_l	0.2914	0.7631	1.3522
c_r	0.3076	0.8042	1.3824
c	0.2995	0.7837	1.3673
$\mu_\theta(c)$	0.8052	0.9049	0.8732
$\mu_\theta^{\mathrm{LMF}}(c)$	0.7400	0.8098	0.7643
$\mu_\theta^{\mathrm{UMF}}(c)$	0.8704	1.0000	0.9821
$p_\theta \times \mu_\theta(c)$	0.1315	0.2240	0.5145
$p_\theta \times \mu_\theta^{\mathrm{LMF}}(c)$	0.1209	0.2004	0.4503
$p_\theta \times \mu_\theta^{\mathrm{UMF}}(c)$	0.1422	0.2475	0.5786
CL		1.0146	
CS		1.0543	
CR		1.0939	

在完成表 3 的计算后，根据表 1 确定方向中心 CS 所在的原子方向，该原子方向即为 A 与 B 的主要方向关系，进而可以利用式(15)—(17)估计 A 与 B 方向关系的不确定性。

5 结 论

方向关系在概念上的模糊性、层次性等特点决定了方向关系表示和推理的复杂性，而现有的模型以简单的方法来进行方向关系的描述和推理。本文首先建立了八方向模糊不均匀划分模型。空间数据固有的不确定性直接影响空间目标间的方向关系，在进行空间关系模糊描述和推理时需要考虑参考点和目标点的不确定性。通过参考点的点位误差可以求解模糊方向隶属度的误差，利用二型模糊理论建立了考虑参考点点位误差的八方向模糊不均匀划分模型，对方向隶属度的不确定性进行了估计，并给出了点与多边形的方向关系确定方法。

利用模糊数学给模糊对象建模时通常带有一定的主观性。二型模糊理论能够描述主隶属度的不确定性，增强了模型的客观性。但是二型模糊集的计算量大，而区间二型模糊集在一定程度上降低了复杂度和计算量。笔者认为利用区间二型模糊理论来进行模糊对象和方向关系的表示更符合实际情况。

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