Numerical study on the effect of multiple scattering on upward scattering coefficients and diffuse absorption coefficients of medium in water

ZHAO Qiao-hua[1], **SONG Yu-zhi**[2]

1. *College of Remote Sensing*, *Nanjing University of Information Science and Technology*, *Jiangsu Nanjing* 210044, *China*;
2. *College of Environmental Science and Engineering*, *Nanjing University of Information Science and Technology*, *Jiangsu Nanjing* 210044, *China*

Abstract: Upward scattering coefficients and diffuse absorption for the down- and up-welling streams are key factors for remote sensing of waters, and the interaction between biology and optics. They are also important parameters of underwater light fields. On the assumption that the radiance field has no internal light source, no inelastic scattering and the water surface was flat, we studied the effects of the zenith of incident, albedo and scattering coefficient on upward scattering coefficient, diffuse absorption coefficient for the downwelling and upwelling streams on condition that a scattering phase function was selected. The upward scattering coefficient showed an increase with the zenith of incident increasing, but the coefficient did not change along with the albedo and scattering coefficient just below water surface. With depth increasing, the profile of upward scattering coefficients gradually approached a constant, and the constant increased with albedo increasing. The profile of upward scattering coefficients strictly increased and then gradually approached a constant with depth increasing when the incidence was normal to the water surface. With further enhancement of zenith angle, the profile gradually increased, then decreased, hereafter kept invariable; while the invariant value increased with albedo increasing, but did not change with zenith angle of incident. The profiles of diffuse absorption coefficients for downwelling streams gradually shifted from strictly increase and followed stability to the new trend which took on first increase and next decrease, then to a constant state. Given that scattering coefficient and albedo was the same, the diffuse absorption coefficients for downwelling streams of different incident zenith converged to stability gradually. The less the albedo was, the more rapid convergent rate was, the shallower the depth of approaching to asymptotic state was. The characteristics of diffuse absorption coefficient for upwelling streams were very similar to that of diffuse absorption coefficient for downwelling streams, the former was merely bigger than the latter, and the diffuse absorption coefficients for upwelling streams were the maximum among them.

Key words: upward scattering coefficient, diffuse absorption for the downwelling streams, diffuse absorption for the upwelling streams, multiple scattering

CLC number: P343.3/TP79 **Document code:** A

1 INTRODUCTION

The transmission and distribution of photosynthesis active radiation (PAR) has an obvious interaction with the water environment. On the one hand, the distribution of PAR not only influences the water's primary productivity, competition for light between the phytoplankton and submerged plants, the balance of carbon dioxide and the cyanobacteria bloom (sea red tide) (Molen *et al*., 1994; Chami & Robilliard, 2002; Woźnika *et al*., 2002, 2003; Arst *et al*., 2008), but also changes the growth of phytoplankton and submerged plant and community structure (Elisabeth *et al*., 2001). On the other hand, light field is the information carrier of water body after scattering and selectively absorbing (Ragni & D'alcalà, 2004), The water-leaving radiance signal composed of upward diffusion scattering is the information source of remote sense for the water body chlorophyll density, the suspension density and so on (Kirk, 1989).

During the light transfers in the water, radiance will experience multiple scattering by particles and selective absorption of the media. Although the scattering does not weaken the energy of PAR in the water, it can actually change the direction of light transmission, change the geometric structure of underwater light field and enlarge the proportion of diffuse underwater light field, and lengthen the light transmission path, so that the probability of the absorbed energy of light increases (Kirk, 1989, 2007). Thus in the process of construction remote sensing

Received: 2008-05-16; **Accepted**: 2009-01-13
Foundation: National natural science foundation(No. 40701168).
First author biography: ZHAO Qiao-hua (1972—), male, associate professor. He received the Ph. D degree in Nanjing Geography & Limnology Institute, Chinese Academy of Science. His research interest includes Remote sensing for water environment, hydro-optics. He authors almost 20 technical papers. E-mail: qhzhao@sina.com

model for waters and biological-optical model, the water's inherent optical parameters (downward and upward absorption coefficient, backscattering coefficient) can not truly represent the capacity for absorbing light energy of water and the proportion of upward transmission of light energy caused by scattering. The estimation of the light energy absorbed by waters should consider the diffuse condition of light fields, otherwise there may be a larger error in the quantitative relationship between the light and the water body's primary productivity (Sathyendranath & Platt, 1989; Kirk, 2007). About quantitative remote sensing of waters, the geometric structure of light field similarly influences the depth of remote sensing for waters (Gordon & McGlunev, 1975). In order to solve the problems above, many scholars devoted to analyzing geometric structure (average cosine) of light field in waters and compute the scalar irradiance (Bannister, 1992; Liu, 2006). In brief, the change of the upward scattering coefficients and the diffuse absorption of waters is the fundamental question in water optics, the remote sensing for waters and the algae photosynthesis.

In this paper, on the assumption that there are no internal light source and no inelastic in waters scattering and the water surface is flat, the effects of the zenith of incident, albedo and scattering coefficients on upward scattering coefficients, down-and upward diffuse absorption were studied on condition that a scattering phase function was selected, to reveal the rule of the effect of different factors on various parameters depth profile, and so as to lay the foundation for further establishing biological-optical models and quantitative model of remote sensing for waters.

2 THEORY AND METHOD

2.1 Upward scattering coefficient

Suppose dF is the upward radiant flux that scattered by diffuse downwelling flux (F) after transmitting dz distances, the upward scattering coefficient (Sathyendranath & Platt, 1991) can be expressed as follows: (for simplicity, we suppress the explicit wavelength dependence of the follow quantities).

$$B_u(z) = \mathrm{d}F(z) < \mu(z) > / F(z)\mathrm{d}z \tag{1}$$

where, B_u denotes the upward scattering coefficient, dz is the vertical distances, and $<\mu(z)>$ is the average cosine at depth z. Weighted by the radiance from zenith, $<\mu(z)>$ can be expressed as:

$$< \mu(z) >= \frac{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} L(\theta,\phi,z)\cos\theta\sin\theta\mathrm{d}\theta\mathrm{d}\varphi}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} L(\theta,\phi,z)\sin\theta\mathrm{d}\theta\mathrm{d}\varphi}. \tag{2}$$

where θ is the zenith angle, ϕ is the azimuth, and $L(\theta, \phi, z)$ is the radiance at depth z. Therefore in order to quantitatively calculate the upward scattering coefficient, we should first compute the upward radiation flux d$F(z)$ which is caused by scattering in the vertical distance dz, the downwelling radiation flux at depth z, as well as average cosine of underwater light field.

2.2 The redistribution of the incident radiation caused by scattering

In order to calculate the upward scattering coefficient, we should accurately know the scattered radiance distribution on different zenith angles for the incidence angle θ.

According to the mode of Sathyendranath and Platt (1991), we firstly define a cone taking the zenith angle θ_f as the axis, while $\theta_1 = \left(\theta_f - \frac{1}{2}\Delta\theta\right)$ and $\theta_2 = \left(\theta_f + \frac{1}{2}\Delta\theta\right)$ as the zenith angles of two elements of the cone, respectively. θ_f is the zenith angle of the scattered radiation, θ is the zenith angle of incident radiation. Suppose χ is the scattering angle, φ_f and φ are the azimuth angles of incident and scattering radiation, we have the relation as follows:

$$\cos\theta_f = \cos\theta\cos\chi + \sin\theta\sin x\cos\psi \tag{3}$$

From (3) we could obtain the range of the scattering angle χ and scattering azimuth ψ ($\psi = [\pi - (\varphi_f - \varphi)]$) as follows (Sathyendranath & Platt, 1991):

$$\begin{aligned} \chi_{\min} &= |\theta_1 - \theta| \\ \chi_{\max} &= \theta_2 + \theta \end{aligned} \tag{4}$$

$$\begin{aligned} \cos\psi_1 &= \frac{\cos\theta_1 - \cos\theta\cos\chi}{\sin\theta\sin\chi} \\ \cos\psi_2 &= \frac{\cos\theta_2 - \cos\theta\cos\chi}{\sin\theta\sin\varphi} \end{aligned} \tag{5}$$

The scattering coefficient on zenith angle θ of the incident radiation by scattering into the direction of zenith angle θ_f can be expressed as $\Re(\theta,\theta_f)$ (Sathyendranath & Platt, 1991):

$$\Re(\theta,\theta_f) = \frac{2}{\Delta\theta}\int_{\chi_{\min}}^{\chi_{\max}} (\psi_1 - \psi_2)\beta(\chi)\sin\chi\mathrm{d}\chi \tag{6}$$

where $\beta(\chi)$ is the volume scattering function.

The upward scattering coefficient $B_u(\theta)$ caused by incident radiation of zenith angle θ after scattering can be expressed as follows:

$$B_u(\theta) = \int_{\frac{\pi}{2}}^{\pi} \Re(\theta,\theta_f)\mathrm{d}\theta_f \tag{7}$$

2.3 The effect of multiple scattering on downward irradiance and the computation of other correlated parameters

For further calculating the variation of upward diffuse scattering coefficient in traversing distance, we suppose a water body with the following inherent optical properties: the absorption coefficient $a(z)$, volume scattering function $\beta(\chi)$, and an incident irradiance $\xi(\theta,z)$ at depth z and zenith angle θ. After the radiation transmitting dz distance, the irradiance $\mathrm{d}\xi(\theta,z+\mathrm{d}z)$ scattered out of the beam can be expressed as (Sathyendranath & Platt, 1991):

$$\mathrm{d}\xi(\theta, z+\mathrm{d}z) = \xi(\theta, z)\exp[-a(z)\mathrm{d}z/\cos\theta]\times \{1-\exp[-b(z)\mathrm{d}z/\cos\theta]\} \tag{8}$$

where, $b(z) = \int_{4\pi} \beta(\chi, z)\mathrm{d}\omega$ is the scattering coefficient, and ω is the solid angle. Let $\mathrm{d}^2\xi(\theta,\theta_\mathrm{f})$ be the portion of flux that is scattered into zenith angle θ_f.

$$\mathrm{d}^2\xi(\theta,\theta_\mathrm{f}, z+dz)\mathrm{d}\theta_\mathrm{f} = \mathrm{d}\xi(\theta, z+\mathrm{d}z)\Re'(\theta,\theta_\mathrm{f}, z)\mathrm{d}\theta_\mathrm{f}, \tag{9}$$

where $\Re'(\theta,\theta_\mathrm{f},z) = \Re(\theta,\theta_\mathrm{f},z)/b(z)$. So irradiance at the zenith angle θ_f may be approximated as follows:

$$\xi(\theta_\mathrm{f}, z+\mathrm{d}z) = \xi(\theta_\mathrm{f}, z)\exp\{-[a(z)+b(z)]\mathrm{d}z/\cos\theta\} + \int_0^{\pi/2} \mathrm{d}^2\xi(\theta,\theta_\mathrm{f}, z+\mathrm{d}z)\mathrm{d}\theta \tag{10}$$

Via transforming $\xi(\theta, z)$ into radiance $L(\theta, z)$, we can obtain $\mu(z)$ by (2). The result compared well with Monte Carlo's result (Sathyendranath & Platt, 1991).

For obtaining the upward scattering coefficient at different depths, the total radiation flux $F(z)$ and upward radiation flux $\mathrm{d}F(z)$ at depth z are needed, and can be calculated by equations (11) and (12).

$$F(z) = \mathrm{d}A\int_0^{\pi} \xi(\theta, z)\mathrm{d}\theta \tag{11}$$

$$\mathrm{d}F(z) = \mathrm{d}A\mathrm{d}z\int_0^{\pi/2} \xi(\theta, z)B_\mathrm{u}(\theta, z)/\cos\theta\mathrm{d}\theta \tag{12}$$

Thus $B_\mathrm{u}(z)$ can be calculated by equation (1).

According to the distribution of the radiance $L(\theta, z)$ at different depths, the average cosines $\mu_\mathrm{d}(z)$, $\mu_\mathrm{u}(z)$ of downward and upward light field as well as the diffuse absorption coefficients $a_\mathrm{d}(z)$, $a_\mathrm{u}(z)$ for the downward and upwelling streams can be expressed as the following equations (Kirk, 1981),

$$a_\mathrm{d}(z) = a(z)/\mu_\mathrm{d}(z) = a(z) \Bigg/ \frac{\int_0^{\pi/2} L(\theta, z)\cos\theta\sin\theta\mathrm{d}\theta}{\int_0^{\pi/2} L(\theta, z)\sin\theta\mathrm{d}\theta} \tag{13}$$

$$a_\mathrm{u}(z) = a(z)/\mu_\mathrm{u}(z) = a(z) \Bigg/ \frac{\int_{\pi/2}^{\pi} L(\theta, z)\cos\theta\sin\theta\mathrm{d}\theta}{\int_{\pi/2}^{\pi} L(\theta, z)\sin\theta\mathrm{d}\theta} \tag{14}$$

2.4 Simulation and calculation

In order to calculate the upward scattering coefficient, firstly, $\Re(\theta,\theta_\mathrm{f})$ is determined by the volume scattering function; and then the distribution of the scattered radiation is calculated. With an condition that θ_i, a_j, b_m are given, firstly, we calculate $\xi(\theta, z)$ at depth z, then calculate the light field re-distribution through absorption and scattering on traversing distance $\mathrm{d}z/\cos\theta$, and finally calculate $B_\mathrm{u}(z)$, $a_\mathrm{d}(z)$, $a_\mathrm{u}(z)$ (Fig. 1). For simplicity, we assume the light properties of the

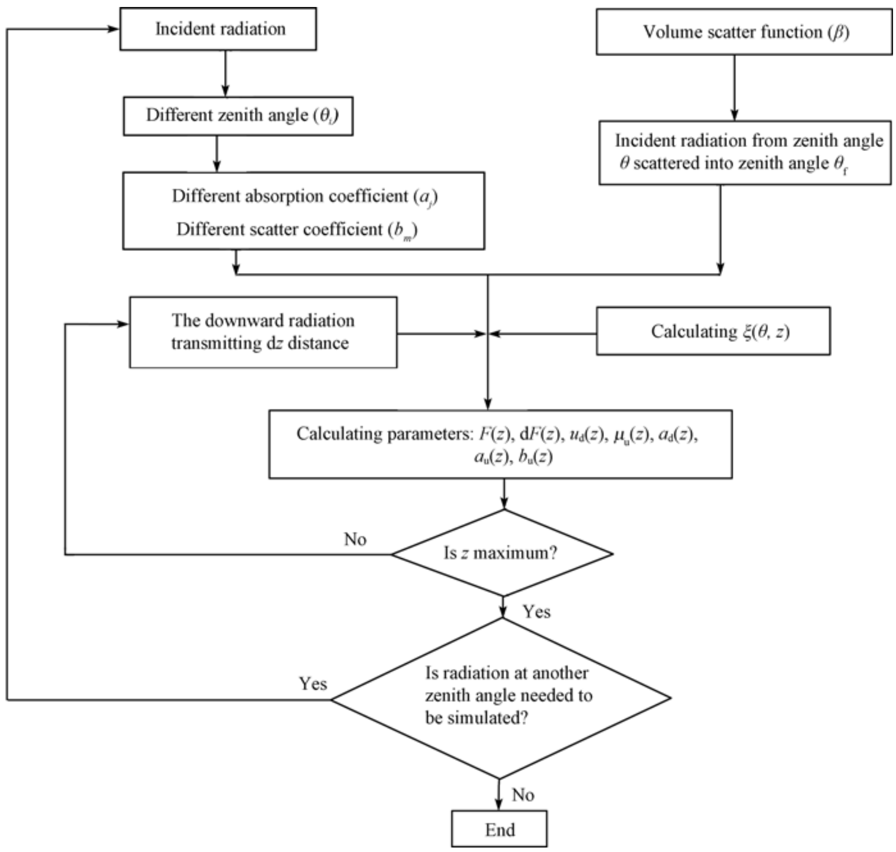

Fig. 1 Schematic diagram of the simulation flow

water is homogeneous; the volume scattering function is the result measured by Morel (Sathyendranath & Platt, 1991); the scattering coefficients are 0.1, 0.3, 0.5, respectively; the albedos are 0.9, 0.7, 0.5, 0.3, respectively; and the incident zenith angles are 0°, 20°, 40°, 60° and 80°, respectively.

3 RESULTS AND ANALYSIS

3.1 Upward diffuse scattering coefficient

The profiles of upward diffusion scattering coefficients in different situations showed the following characteristics:

(1) The upward scattering coefficients increase with the zenith angle of incident just below water surface, but do not change with albedos and scattering coefficients (Fig. 2). These are mainly because the transmission direction of incident radiation does not change just after entering the water. When the incident zenith angle is 0^{o}, the upward diffuse scattering is mainly determined by the backscattering coefficient which is inherent optical parameters; and therefore, the upward diffuse scattering coefficient is very small. With the zenith angle of incident radiation increasing, a part of back-scattering rays replaces the forward scattering rays and the upward scattering flux includes part of forward scattering flux, so the proportion of upward scattering flux rises. Furthermore, the upward diffuse radiation increases and the total irradiance reduces, so the upward scattering increases with the incident zenith angle increasing. The absorption and the scattering have not manifested their effects on the incident radiation for it has just entered the water, and the albedo and scattering coefficient have not shown any influence on it.

(2) The profiles of upward scattering coefficients gradually approach a constant with depth increasing, and the value of the constant increases with albedo increasing (Fig. 2). It may be because the underwater radiation gradually gets close to an asymptotic state after entering the water by scattering and absorption, and then makes the incident zenith angle and other environmental information disappear, showing a characteristic of asymptotic light field (Piening & McCormick, 2003), so the radiation which is close to an asymptotic state takes on the characteristics of inherent optical property. And the effect of scattering becomes more significant with the albedo increasing, which results in the upward scattering coefficient increasing close to an asymptotic state. On the contrary, the proportion of diffuse light will be restrained with the absorption strengthening, thus its upward diffuse scattering coefficient becomes small correspondingly.

(3) Upward diffuse scattering coefficients gradually increase strictly with the depth at first, then levels off when the incidence is normal. But with the incident zenith angle increasing, Upward diffuse scattering coefficients first increases and then decreases, finally approaches an asymptotic state; and the more albedo, the bigger incident zenith angle which is needed to take on this kind of phenomenon (Fig. 2). It is mainly because the incident radiation will be affected by the scattering and the absorption during the transmission in the water. When the incident radiation is normal to water surface, the radiation continuously undergoes the scattering effect, so that the diffuse proportion in waters increases, and more radiation deviates from the original transmission direction. More and more forward scattering turns into upward scattering, and thus upward scattering coefficient increases gradually. However, with the proportion of diffuse increasing continuously, the distance of photon transmitting extends, and the absorbed possibility would grow. Absorption reduces the proportion of diffuse, and indirectly reduces the upward scattering coefficient. Thus, with the radiation transmitting downward, the upward diffuse scattering coefficients begin to increase, then are restrained, and finally reach an asymptotic state (Kirk, 1989, 1999, 2007). When the incident zenith angle increases, in traversing the vertical distance dz, even along the direction of incident, the light traversing distance extends on account of scattering, so that the ratio of diffusion is similar to be added on a certain cardinal number. When the surface's upward diffuse scattering coefficient is greater than that at the asymptotic state, the absorption will reduce this coefficient with depth increasing. It can be seen that the profile of upward scattering coefficient can present the non-monotonous phenomenon only with a large zenith angle.

3.2 Diffuse absorption coefficient for the downward and up welling streams

Fig.3 shows the profiles of diffuse absorption coefficient for the downwelling streams in conditions of different scattering coefficients, albedo and zenith angles.

The profiles take on three typical features:

(1) The profile gradually increases at first, and then decrease with zenith angle increasing, which indicates that the downward average cosine decreases with depth at small zenith angles, then shifts to increase with depth when zenith angle increasing (Fig. 3).

(2) When the scattering coefficient and albedo are the same, the diffuse absorption coefficients for the downwelling streams with different zenith angles gradually converge on a same stable state. The less the albedo is, the more rapid the convergent rate is, and the shallower the depth of approaching to the asymptotic state is. It shows that the absorption and scattering can be easy to achieve equilibrium when the former is stronger and the latter is weaker (Fig. 3).

(3) The diffuse absorption coefficient for the downwelling streams is always bigger than the absorption coefficient which is an inherent optical property (Fig. 3).

The characteristics of the diffuse absorption coefficient for upwelling streams are very similar to the ones of diffuse absorption coefficient for downwelling streams (Fig. 4). The main difference is that diffuse absorption coefficients for upwelling streams are always bigger than the ones for downwelling streams in the same situation. The reason is that the upward radiation raises from the downward radiation by scattering, so the upward light field's average cosine is smaller than the downward light field's, which is consistent with the result of Kirk (1981). The big diffuse absorption coefficient for upwell-

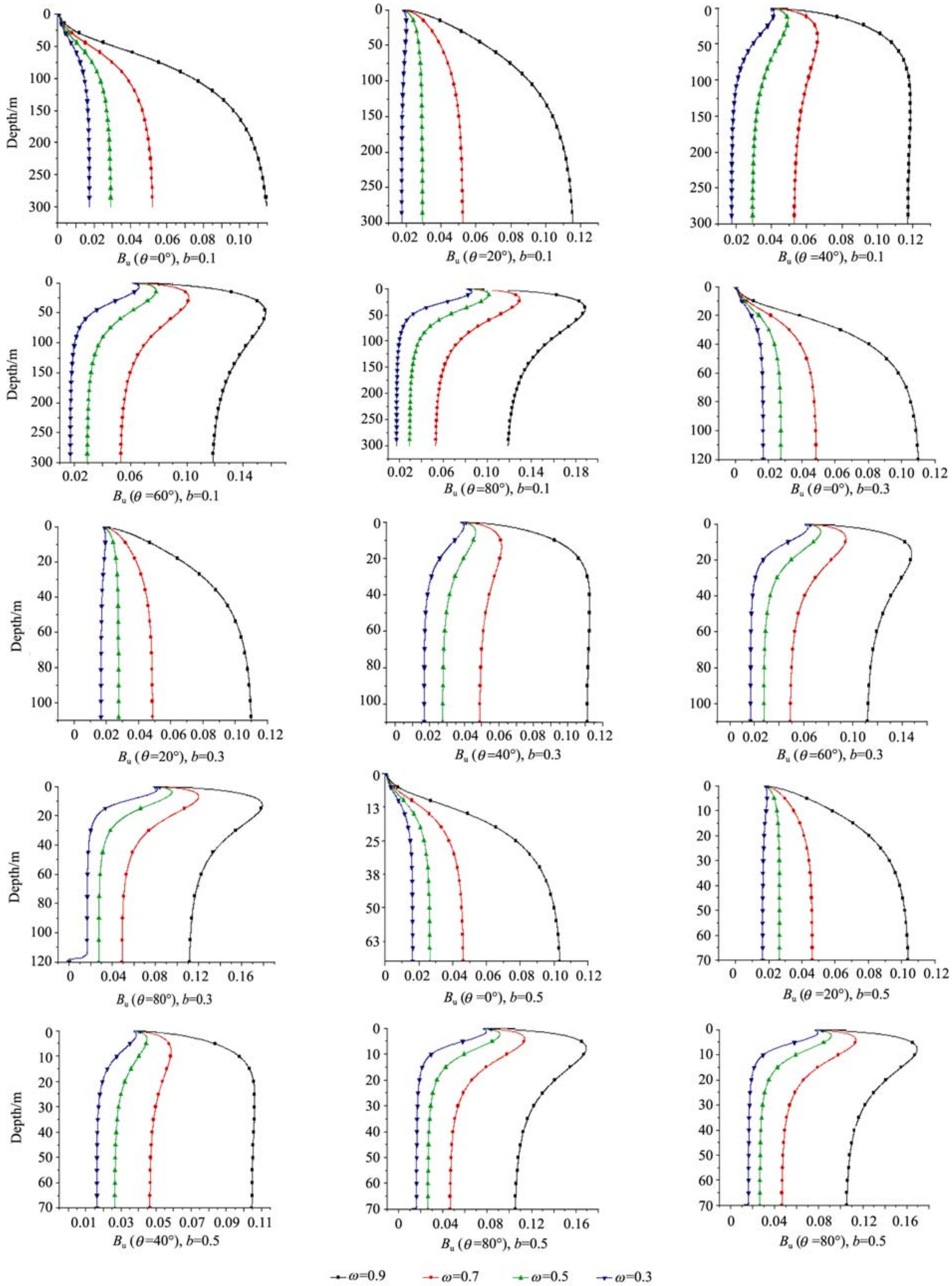

Fig. 2 The profiles of upward scattering coefficients in conditions of different scattering coefficients, albedo and zeniths

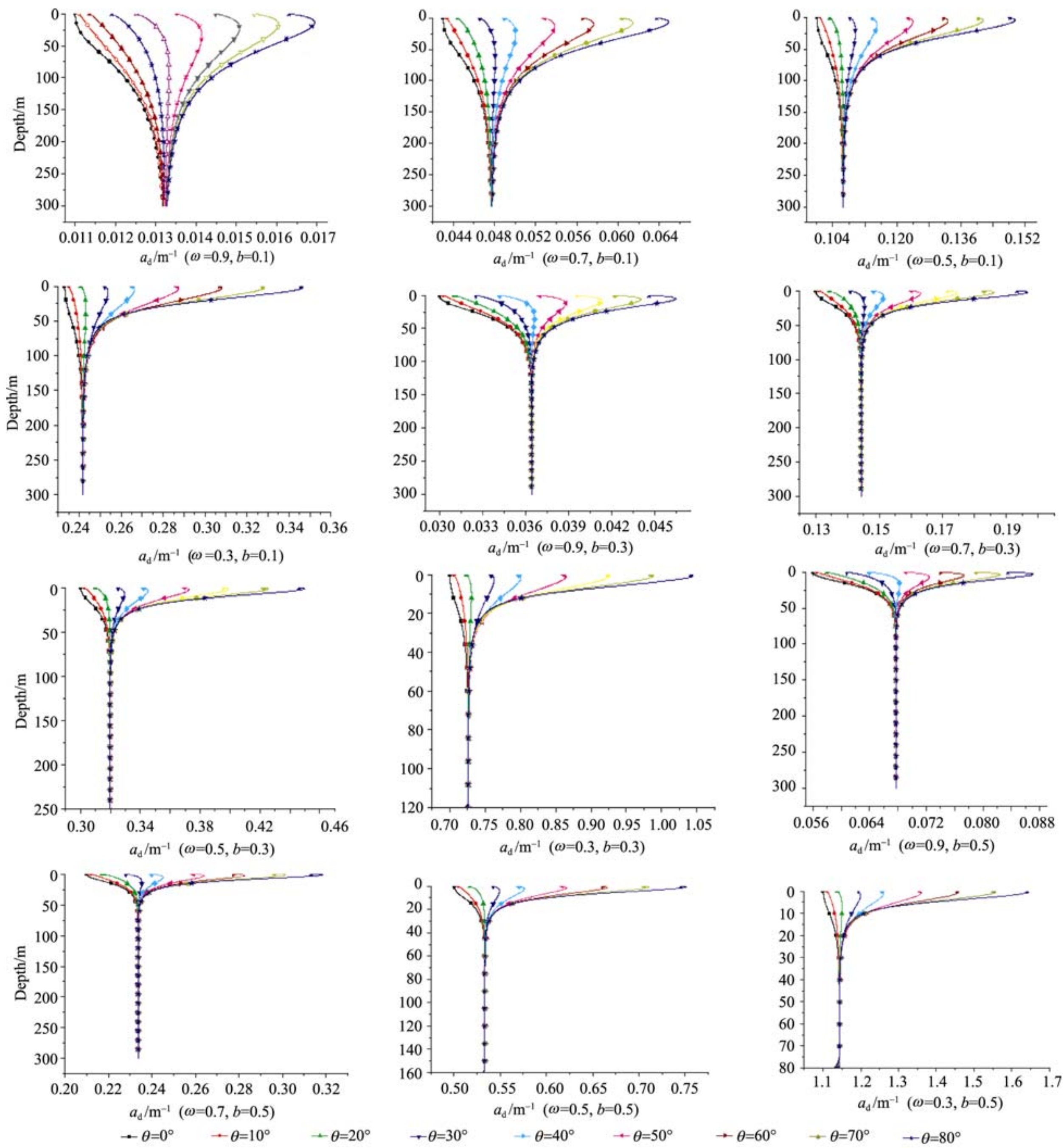

Fig. 3 The profiles of downward absorption coefficients in conditions of different scattering coefficients, albedos and zeniths

ing streams results in diminution of the penetrating rate of upward radiation. This also coincides with the fact that the water-leaving radiation is small in the water body and the satellite sensor receives very slightly signal from water.

4 DISCUSSION

When the radiation is traversing in the water, the absorption action can not change the direction of radiant, but reduce the radiation energy. The scattering is on the contrary. With the traversing path extending and the number of scattering events increasing, more and more radiation deviates from the original direction, and the average cosine of light field reduces gradually. For a pure scattering water, the light field average cosine (μ) can be denoted as follows (Kirk, 1989, 1999).

$$\mu = \mu_0 \exp[-bd(1-\mu_s)] \quad (15)$$

where b is the scattering coefficient, d is photon's total traversing distance, μ_s is the average cosine of scattering. From equation (15) we can see that for pure scattering medium, if the photon's traversing distance and the scattering coefficient are large, the light field's average cosine will be small, and then the diffusion proportion will be high. According to Mie-scattering theory, the bigger the zenith angle of downward radiation, the more proportion of forward scattering becomes upward transmission, and the bigger the upward diffuse scattering coefficient.

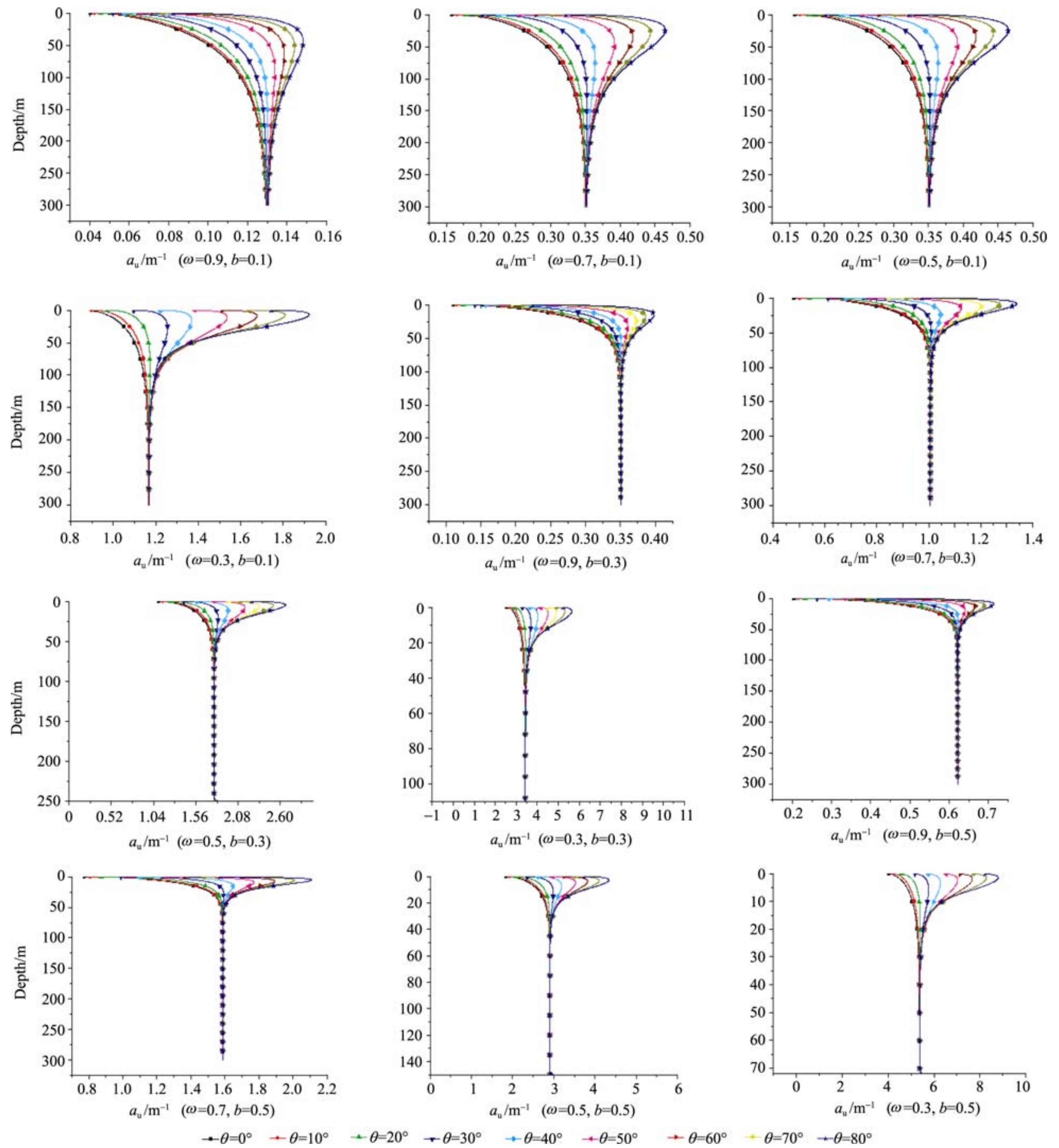

Fig. 4 The profiles of upward absorption coefficients in conditions of different scattering coefficients, albedos and zeniths

Although the absorption does not change the direction of the photon's transmission, it may influence the geometric structure of underwater light field. In the water body, the absorption can be expressed as (Sathyendranath & Platt, 1991; Kirk, 1999):

$$N = N_0(1-\exp(-aC_{\mathrm{w}}t), \qquad (16)$$

where N is the photon number been absorbed, N_0 is the initial photon number, C_{w} is the speed of the photon transmitting in the water, and t is the photon's transmission time in the water. We can see that with the photon's transmission time extending, the probability of the photons scattered increasing and the photon's course lengthening, the probability of the photons absorbed increases. That means the bigger the zenith angle, the more photons will be absorbed. When the two actions achieve balance, the upward scattering coefficients and diffuse absorption coefficients for the down- and upwelling streams also reach a balanced state, which is named asymptotic state (Piening & McCormick, 2003) (Fig. 2, 3, 4). Thus, we can explain that so long as the incident zenith angle is the same, the upward diffuse scattering coefficient is almost equivalent when the radiation just entering/ has just entered the water (Fig. 2). Similarly we can explain that the more the albedo and the less the absorption effect relatively, the more upward diffuse scattering coefficient when underwater light field is approaching to an asymptotic state (Fig. 2). Thus, the characteristic in Fig. 3 and 4 can also get explanation completely.

During the process of photons traversing downward, the proportion of the incident photons which can neither be absorbed nor be scattered reduces with extension of the traversing distance, so that the photons turn less and less when traversing along the initial direction of the radiation. When the photon number becomes equal in all directions, that is the time of asymptotic state of light field appearing, the apparent optical properties are affected not by water roughness, atmospheric conditions or incidence zenith angle, but by only the optical properties of waters. Therefore, the apparent optical property becomes the inherent optical property (Leathers & McCormick, 1997).

The upward traversing is caused by the scattering, and its radiation spatial distribution totally depends on the volume scattering function of medium. Therefore, the upward radiation is easier absorbed, and the attenuation coefficient of upward diffusion is big, so that the water leaving radiance is generally rather small.

Because of the interactive actions of scattering and absorption, when estimating the primary productivity and photosynthesis of water body, we can not simply calculate the inherent optical parameters and downward irradiance. Geometric structure of the underwater light field should also be taken into consideration.

5 CONCLUSION

The upward scattering coefficient showed an increase with the incident zenith angle increasing, but the coefficient did not change along with the albedo and scattering coefficient just below water surface. With depth increasing, the profile of upward scattering coefficients gradually approached a constant, and the constant increased with albedo increasing. The profile of upward scattering coefficients strictly increased and then gradually approached a constant with depth increasing when the incidence was normal to the water surface. With further enhancement of zenith angle, it happened that the profile gradually increased, then decreased, hereafter kept invariable. The invariant value increased with albedo increasing, but did not change with zenith angle of incident. The profiles of diffuse absorption coefficients for downwelling streams gradually shifted from strict increase and following stability to a new trend that they first increased and next decreased and then to a constant state.

When the scattering coefficient and albedo were the same, the diffuse absorption coefficients for downwelling streams of different incident zenith converge to stability gradually. The less the albedo was, the more rapid convergent rate was, the shallower the depth of approaching to asymptotic state was. The characteristics of diffuse absorption coefficient for upwelling streams were very similar to the ones of diffuse absorption coefficient for downwelling streams; while the former was merely bigger than the latter, and the diffuse absorption coefficients for upwelling streams were the maximum among them.

REFERENCES

Arst H, Nõges T, Nõges P and Paavel B. 2008. Relation of phytoplankton in stiu primary production, chlorophyll concentration and underwater irradiance in turbid lakes. *Hydrobiologia*, **599**: 169—176

Bannister T T. 1992. Model of the mean cosine of underwater radiance and estimate of underwater scalar irradiance. *Limnol*. *Oceanogr*, **37**(4): 773—780

Chami M and Robilliard D. 2002. Inversion of oceanic constitute in case I and II waters with genetic programming algorithms. *Applied Optics*, **41**(30): 6260—6274

Elisabeth H S, Duin V, Bloom G, Robert Maffione, Richard Zimmerman, Carl F Cerco, Mark Dortch and Elly P.H. Best. 2001. Modeling underwater light climate in relation to sedimentation, resuspension, water quality and autotrophic growth. *Hydrobiologia*, **444**: 25—42

Gordon H R and McGlunev W R. 1975. Estimation of the depth of sunlight penetration in the sea for remote sensing. *Applied Optics*, **14**: 413—416

Kirk J T O. 1981. Monte Carlo study of the nature of the underwater light field in,and the relationships between optical properties of, turbid yellow waters. *Aust.J.Mar.Freshwater Res*, **32**: 517—532

Kirk J T O. 1989. The upwelling light stream in natural waters. *Limnol. Oceanogr*, **34**(8): 1410—1425

Kirk J T O. 1999. Multiple scattering of a photon flux: implications for the integral average cosine of the underwater light field. *Applied Optics*, **38**(15): 3134—3140

Kirk J T O. 2007. Radiation transfer in ocean water: Average penetration depth of a photon flux in a fixed time interval. *Journal of Geophysical Research*, c02010, doi: 10. 1029/2006JC003500

Leathers R A. and McCormick N J. 1997. Ocean inherent optical property estimation from irradiances. *Applied Optics*, **36**(33): 8685—8698

Liu C C. 2006. Fast and accurate model of underwater scalar irradiance for stratified case 2 waters. *Optics Express*, **14**(5): 1703—1719

Molen D T V D, Los F J and Ballegooigen L V and Vat M V D. 1994. Mathematical modeling as a tool for management in eutrophication aontrol of shallow lakes. *Hydrobiologia*, **275/ 276**: 479—492

Piening B D and McCormick N J. 2003. Asymptotic optical depths in source-free ocean waters. *Applied Optics*, 42, 5382—5387

Ragni M and D'alcalà M B. 2004. Light as an information carrier underwater, *Journal of Plankton Research*, **26**(4): 433—443

Sathyendranath S and Platt T. 1989. Computation of aquatic primary production: Extended formalism to include effect of angular and spectral distribution of light. *Limnol. Oeanogr*, **34**(1): 188—198

Sathyendranath S and Platt T. 1991. Angular distribution of the submarine light field: modification by multiple scattering. *Pro. R. Soc. Lond. A*, **433**: 287—297

Woźnika B, Dera J, Ficek D, Ostrowska M and Majchrowski R. 2002, Dependence of the photosynthesis quantum yield in ocean on environmental factors. *Oceanologia*, **44**(4): 439—459

Woźnika B, Dera J, Ficek D, Majchrowski R, Ostrowska M and Kaczmarek S. 2003. Modeling light and photosynthesis in the marine environment. *Oceanologia*, **45**(1): 171—245

多次散射对向上漫射散射系数及漫射吸收系数影响的数值研究

赵巧华[1]，宋玉芝[2]

1. 南京信息工程大学 遥感学院，江苏 南京 210044;
2. 南京信息工程大学 环境科学与工程学院，江苏 南京 210044

摘 要： 在假定水体表面为镜面、水体中无内光源及非弹性散射的情况下、利用多次散射理论，研究了在固定体散射函数的情况下，不同的入射天顶角、单次散射反照率和散射系数对向上漫射散射系数，上、下行漫射吸收系数的影响。结果表明：在紧贴水表的下层，向上漫射散射系数随入射辐射天顶角的增大而增大，但不随单次散射反照率和散射系数的改变而改变；当入射辐射为垂直入射时，向上漫射散射系数的深度廓线随深度呈现单调增的趋势，并逐步接近一个恒定状态；随着入射天顶角的增大，该单调现象发生了变化，出现先增大后减小，再逼进恒定状态，且这个恒定值随单次反照率增大而增大。下行漫射吸收系数随着入射天顶角的增大，其廓线由单调增加过渡到先增大后减小；在散射系数和单次散射反照率相同的条件下，不同入射天顶角的下行漫射吸收系数逐渐收敛于同一稳定的状态。且单次散射率越小，这种收敛的速度越快，达到恒定状态的深度也越浅。上行漫射吸收系数的主要特征基本与下行漫射吸收系数类似，而且两个恒定值均大于属于固有光学参数的吸收系数，其中向上漫射吸收系数最大。

关键词： 向上漫射散射系数，下行漫射吸收系数，上行漫射吸收系数，多次散射

中图分类号： P343.3/TP79 **文献标识码：** A

1 引 言

水体中光合有效辐射的传输及分布与水环境存在明显的相互作用，一方面，光合有效辐射在水体中的分布不仅是影响水体中的初级生产力、浮游植物和沉水植物的竞争、二氧化碳的收支平衡状况、蓝藻水华(海洋赤潮)暴发及其时空变化的主要因子(Molen 等, 1994; Chami & Robilliard, 2002; Woźnika 等, 2002, 2003; Arst 等, 2008)，也是影响浮游植物、沉水植物生长及其群落结构的关键因素(Elisabeth 等，2001)；另一方面，光场经过水体介质的散射和选择性吸收作用后，成为水体信息的携带者(Ragni & D'alcalà, 2004)。介质散射作用引起的向上漫射散射组成的离水辐射信号，是遥感水体叶绿素浓度、悬浮物浓度等的信息源(Kirk, 1989)。

光在水体传输过程中，需经历颗粒物的多次散射及介质选择性吸收的作用，尽管散射并未使水体中光合有效辐射的能量减小，但能改变光的传输方向，改变水体中光场的几何结构，使得水体光场中漫射的比例增大，光的传输路径加长，光能被吸收的概率随之增大(Kirk，1989，2007)，因而在构建水体遥感、生物-光学模型过程中，水体的固有光学参数(上、下吸收系数，后向散射系数)并不能真实反映水体对光能的吸收能力及散射引发的光能向上传输的比例。因而在估算水体介质对光能的吸收时，须耦合水体光场的漫射状态，否则会导致光与水体初级生产力的定量关系出现较大误差(Kirk，2007，Sathyendranath & Platt，1989)。在水体定量遥感中，光场的几何结构同样影响遥感对水体探测的深度(Gordon & McGlunev, 1975)。为了解决上述问题，诸多学者致力于研究水体中光场的几何结构(平均余弦)以及标量辐照度的计算(Bannister，1992；Liu，2006)。可见向上漫射散射系数，上、下行漫射吸收系数的变化规律是水光学、水体遥感及藻类光合作

收稿日期：2008-05-16；修订日期：2009-01-13

基金项目：国家自然科学基金项目“大型浅水湖泊中藻类光利用效率的时空变化规律研究”(编号：40701168)及中国科学院南京地理与湖泊研究所湖泊与环境国家重点实验室开放基金。

第一作者简介：赵巧华(1972—)，男，博士，2007 年毕业于南京地理与船泊研究所，目前主要从事水光学及大气光学的研究。qhzhao@sina.com。

用中的基础问题。

2 理论与方法

2.1 向上漫射散射系数

假定下行漫射辐射通量(F)向下传输一垂直距离微元 dz, 由于散射作用引起的向上辐射通量为 dF, 则向上漫射散射系数(Sathyendranath & Platt, 1991)可用下式表示: (为简化, 各式中省略波长参数, 下同。)

$$B_{\rm u}(z)={\rm d}F(z)<\mu(z)>/F(z){\rm d}z \tag{1}$$

式中, $B_{\rm u}$为向上漫射散射系数; $<\mu(z)>$为深度 z 处的光场平均余弦, 可表示为:

$$<\mu(z)>=\frac{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}L(\theta,\phi,z)\cos\theta\sin\theta{\rm d}\theta{\rm d}\varphi}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}L(\theta,\phi,z)\sin\theta{\rm d}\theta{\rm d}\varphi}. \tag{2}$$

式中, θ为天顶角, ϕ为方位角, $L(\theta,\phi,z)$为深度 z 处的辐亮度。要定量计算向上漫射散射系数, 需先定量计算在 dz 的垂直距离上, 由散射作用引起的向上漫射散射通量 d$F(z)$、深度为 z 处的下行辐射通量 $F(z)$以及光场平均余弦$<\mu(z)>$。

2.2 散射引起入射辐射的重新分布

为了定量计算向上漫射散射系数, 应准确知道入射角为θ的辐亮度, 发生散射后, 在各个天顶角方向上的重新分布状况。

根据 Sathyendranath 和 Platt(1991)的模式, 先定义一个以天顶角$\theta_{\rm f}$ 为轴线的圆锥体, 其母线的天顶角范围分别为 $\theta_1=\left(\theta_{\rm f}-\frac{1}{2}\Delta\theta\right)$ 和 $\theta_2=\left(\theta_{\rm f}+\frac{1}{2}\Delta\theta\right)$, 并以此作为散射的天顶角。且以$\theta$为入射辐射的天顶角, $\theta_{\rm f}$为散射辐射的天顶角, χ为散射角, $\varphi_{\rm f}$, φ为别为入射和散射辐射的方位角。则它们之间存在如下的关系:

$$\cos\theta_{\rm f}=\cos\theta\cos\chi+\sin\theta\sin x\cos\psi \tag{3}$$

从(3)式可以得到散射角χ及散射方位角ψ ($\psi=[\pi-(\varphi_{\rm f}-\varphi)]$)的取值范围是(Sathyendranath & Platt, 1991):

$$\begin{aligned}\chi_{\min}&=|\theta_1-\theta|\\ \chi_{\max}&=\theta_2+\theta\end{aligned} \tag{4}$$

$$\begin{aligned}\cos\psi_1&=\frac{\cos\theta_1-\cos\theta\cos\chi}{\sin\theta\sin\chi}\\ \cos\psi_2&=\frac{\cos\theta_2-\cos\theta\cos\chi}{\sin\theta\sin\varphi}\end{aligned} \tag{5}$$

天顶角为θ 的入射辐射经散射进入天顶角为$\theta_{\rm f}$方向散射系数可以表示为$\Re(\theta,\theta_{\rm f})$ (Sathyendranath & Platt, 1991):

$$\Re(\theta,\theta_{\rm f})=\frac{2}{\Delta\theta}\int_{\chi_{\min}}^{\chi_{\max}}(\psi_1-\psi_2)\beta(\chi)\sin\chi{\rm d}\chi \tag{6}$$

式中, $\beta(\chi)$为体散射函数。

入射天顶角θ 的入射辐射由于散射作用引起的向上散射系数 $B_{\rm u}(\theta)$ 可以表示如下:

$$B_{\rm u}(\theta)=\int_{\frac{\pi}{2}}^{\pi}\Re(\theta,\theta_{\rm f}){\rm d}\theta_{\rm f} \tag{7}$$

2.3 多重散射对下行辐照度的作用及几个参数的计算

为进一步计算辐照度在传输过程中向上漫射散射系数等参数的变化, 给定任一水体, 吸收系数为 $a(z)$, 体散射函数为 $\beta(\chi)$, 且深度为 z 处天顶角为θ 的入射辐照度为 $\xi(\theta,z)$, 当该辐照度在垂直方向上传输 dz 距离时, 发生散射的辐照度可表示为(Sathyendranath & Platt, 1991):

$$\begin{aligned}{\rm d}\xi(\theta,z+{\rm d}z)=\xi(\theta,z)\exp[-a(z){\rm d}z/\cos\theta]\times\\ \{1-\exp[-b(z){\rm d}z/\cos\theta]\}\end{aligned} \tag{8}$$

其中, $b(z)=\int_{4\pi}\beta(\chi,z){\rm d}\omega$ 为散射系数, ω为立体角。在这部分发生散射作用的辐照度中, 散射后其天顶角为$\theta_{\rm f}$的辐照度为 ${\rm d}^2\xi(\theta,\theta_{\rm f})$。

$${\rm d}^2\xi(\theta,\theta_{\rm f},z+dz){\rm d}\theta_{\rm f}={\rm d}\xi(\theta,z+{\rm d}z)\Re'(\theta,\theta_{\rm f},z){\rm d}\theta_{\rm f}, \tag{9}$$

其中, $\Re'(\theta,\theta_{\rm f},z)=\Re(\theta,\theta_{\rm f},z)/b(z)$, 则在$\theta_{\rm f}$ 方向上的辐照度可近似为:

$$\begin{aligned}\xi(\theta_{\rm f},z+{\rm d}z)=\xi(\theta_{\rm f},z)\exp\{-[a(z)+b(z)]{\rm d}z/\cos\theta\}+\\ \int_0^{\pi/2}{\rm d}^2\xi(\theta,\theta_{\rm f},z+{\rm d}z){\rm d}\theta\end{aligned} \tag{10}$$

$\mu(z)$可以利用$\xi(\theta, z)$转化为辐亮度 $L(\theta, z)$, 再用式(2)计算得到, 且该参数的计算结果与 Monte Carlo 模拟的结果进行比较, 非常吻合(Sathyendranath & Platt, 1991)。

为得到各深度上的向上漫射散射系数, 须先确定深度 z 处的总辐射通量 $F(z)$和向上的辐射通量 d$F(z)$, 它们可以分别表示为:

$$F(z)={\rm d}A\int_0^{\pi}\xi(\theta,z){\rm d}\theta \tag{11}$$

$${\rm d}F(z)={\rm d}A{\rm d}z\int_0^{\pi/2}\xi(\theta,z)B_{\rm u}(\theta,z)/\cos\theta{\rm d}\theta \tag{12}$$

$B_{\rm u}(z)$可以根据(1)式计算得到。

根据得到各层的辐亮度 $L(\theta, z)$的分布，则上、下行光场的平均余弦 $u_d(z)$、$\mu_u(z)$和上、下行漫射吸收系数 $a_d(z)$、$a_u(z)$可表示如下(Kirk, 1981):

$$a_d(z)=a(z)/\mu_d(z)=a(z)\bigg/\frac{\int_0^{\pi/2}L(\theta,z)\cos\theta\sin\theta\mathrm{d}\theta}{\int_0^{\pi/2}L(\theta,z)\sin\theta\mathrm{d}\theta} \tag{13}$$

$$a_u(z)=a(z)/\mu_u(z)=a(z)\bigg/\frac{\int_{\pi/2}^{\pi}L(\theta,z)\cos\theta\sin\theta\mathrm{d}\theta}{\int_{\pi/2}^{\pi}L(\theta,z)\sin\theta\mathrm{d}\theta} \tag{14}$$

2.4　模拟计算流程

为计算向上漫射散射系数，首先需要根据体散射函数确定 $\Re(\theta,\theta_f)$，其目的是为计算任一方向的入射辐射在经过散射后的空间分布；然后根据在给定 θ_i, a_j, b_m 的情况下，计算 z 深度上的 $\xi(\theta, z)$，再计算经过 dz 距离后吸收、散射作用造成光场的重新分布，最后计算出 $B_u(z)$、$a_d(z)$、$a_u(z)$(图 1)。我们设定水体光学均匀，体散射函数是 Morel 测定的结果(Sathyendranath & Platt, 1991)，散射系数分别为 0.1, 0.3, 0.5；单次散射反照率分别为 0.9, 0.7, 0.5, 0.3；入射天顶角分别为 0°, 20°, 40°, 60°和 80°。

3　结果与分析

3.1　向上漫射散射系数

针对不同的散射系数、单次散射反照率及入射天顶角，进行了说明(图 2)。

向上漫射散射系数的廓线呈现如下特点：(1)从图 2 中各图 0¯m 深度的向上漫射散射系数可以看出,

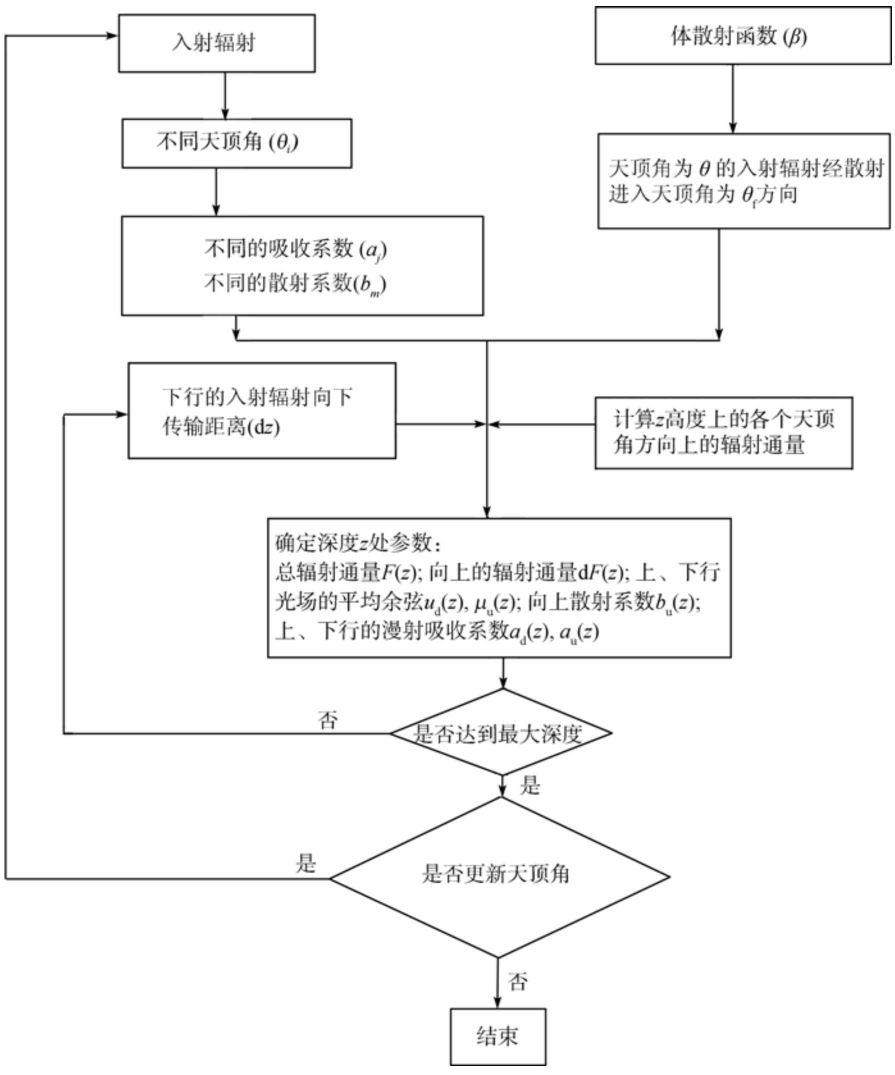

图 1　模拟程序流程示意图

在紧贴水表的下层，向上漫射散射系数随入射辐射天顶角的增大而增大，但不随单次散射反照率和散射系数的改变而改变。这主要是由于入射辐射在刚进入水表后，基本未改变原来的传输方向，因而当入射天顶角为 0°时，向上漫射散射主要由固有光学参数中的后向散射系数决定，所以在该入射天顶角时，其向上漫射散射系数非常小。但随着入射辐射天顶角的增大，通过一部分后向散射对前向散射的置换，向上散射的辐射中含有一部分前向散射，使得向上散射的比例提高；另一方面，由于入射天顶角的增大，其总的辐照度相应减小，表现出向上漫射辐射随天顶角的增大而增大。由于入射辐射刚刚进入水面，吸收和散射对其的作用没有得到明显的体现，所以并未呈现出受单次散射反照率和散射系数变化的影响。(2)向上漫射散射系数的深度廓线随深度的增加，逐渐逼近于一恒定的值，该值随单次散射反照率的增大而增大，图 2 中的各图均存在这种规律。这种现象来源于入射辐射在进入水体后，在散射和吸收的共同作用下，逐渐达到一种平衡状态，使得入射辐射的天顶角等环境信息基本消失，体现出水体中的固有光学特性，呈现出渐进光场的特点(Piening & McCormick, 2003)。单次散射反射率越大，水体中散射作用越强，使得恒定状态下的向上漫射散射系数增大。反之，吸收作用增强，水体中漫射比例将得到抑制，其向上漫射散射系数也相应变小。(3)在入射辐射为垂直入射时，向上漫射散射系数的深度廓线随深度增加呈现单调增的趋势，并逐步进入一个恒定状态。但随着入射天顶角的增大，该单调现象发生了变化，出现先增大后减小，再逼进恒定状态，且单次反照率越大，出现这种现象需要的入射天顶角越大(图 2)。这种现象主要是由于当入射辐射进入水体后，在传输过程中需经历介质散射和吸收的双重作用。当入射辐射天顶角为 0°时，辐射不断经历散射作用，使得水体中的漫射成分比例提高，更多的辐射偏离原来的传输方向，前向散射转向向上传输的比例提高，因而向上漫射散射系数逐渐在增大。但随着漫射比例的继续提高，光子所经过的路程增大，其被吸收的概率也就变大，且漫射比例越高，被吸收的概率也就越大。吸收作用减少了漫射的比例，间接地使向上漫射散射系数减小。因此，随着辐射向下传输，向上漫射散射系数开始呈现增大，随后得到抑制，最后逐渐变成恒定状态(Kirk, 1989, 1999, 2007)。当入射天顶角增大时，在垂直距离 dz 中，即使沿入射辐射方向，光传输的路程也增大，加上在传输过程中的散射作用，使得漫射比例类似于叠加在一定基数上，当表层向上漫射散射系数大于稳定状态的值，则随深度的增加，吸收作用致使该系数减小，呈现出非单调的现象，由此可知，只有在较大的入射天顶角时才会出现向上散射系数廓线的非单调现象。

3.2 上、下行漫射吸收系数

图 3 分别给出了在不同散射系数、不同单次散射反照率和不同入射天顶角情况下的上、下行漫射吸收系数的深度廓线。

下行漫射吸收系数的深度廓线呈现 3 个典型的特征，(1) 随着入射天顶角的增大，廓线由单调增加过渡到先增大后减小。这说明了下行光场平均余弦在小入射天顶角时呈现单调减小的特征，而在大的天顶角时却呈现先减小后增大的特征(图 3)。(2) 在散射系数和单次散射反照率相同的条件下，不同入射天顶角的下行漫射吸收系数逐渐收敛于同一稳定的状态，且单次散射率越小，这种收敛的速度越快，达到恒定状态的深度也越浅。说明在吸收作用相对较强、散射作用相对较弱的情况下，二者的作用相对较易达到平衡状态(图 3)。(3) 下行漫射吸收系数总是大于作为固有光学参数的吸收系数。

上行漫射吸收系数的深度廓线分布规律基本与下行漫射吸收系数一致(图 4)，二者主要区别在于在相同的情况下，上行漫射吸收系数总是大于下行漫射吸收系数。造成这一现象的原因是由于上行辐射是由下行辐射经散射作用而产生的，使得上行光场的平均余弦比下行光场的平均余弦更小，这与 Kirk(1981)的结果吻合。正是由于上行漫射吸收系数较大，从而造成上行辐射的透过率较低，这也与水体中离水辐射小、卫星传感器接收到的水体信号非常小等情况相吻合。

4 讨 论

辐射在水体内传输过程中，吸收作用不改变光的传输方向，只是改变辐射能量的大小。散射改变光的传输方向，不改变能量的大小。随着传输路径的延长、发生散射次数的增加，光场中越来越多的辐射偏离原来的方向，光场的平均余弦逐渐减小。对于一个纯散射的水体，光场的平均余弦(μ)可表示为(Kirk, 1989, 1999):

$$\mu = \mu_0 \exp[-bd(1-\mu_s)] \tag{15}$$

式中，b 为散射系数，d 为光子的总行程，μ_s 为散射平均余弦。从上式看出，对纯散射介质而言，光子的行程和散射系数越大，光场平均余弦越小，则水体中光场的漫射比例就会越高，根据米散射理论，向下辐射的天顶角越大，则在前向散射中向上传输的比例越大，向上漫射散射系数越大。

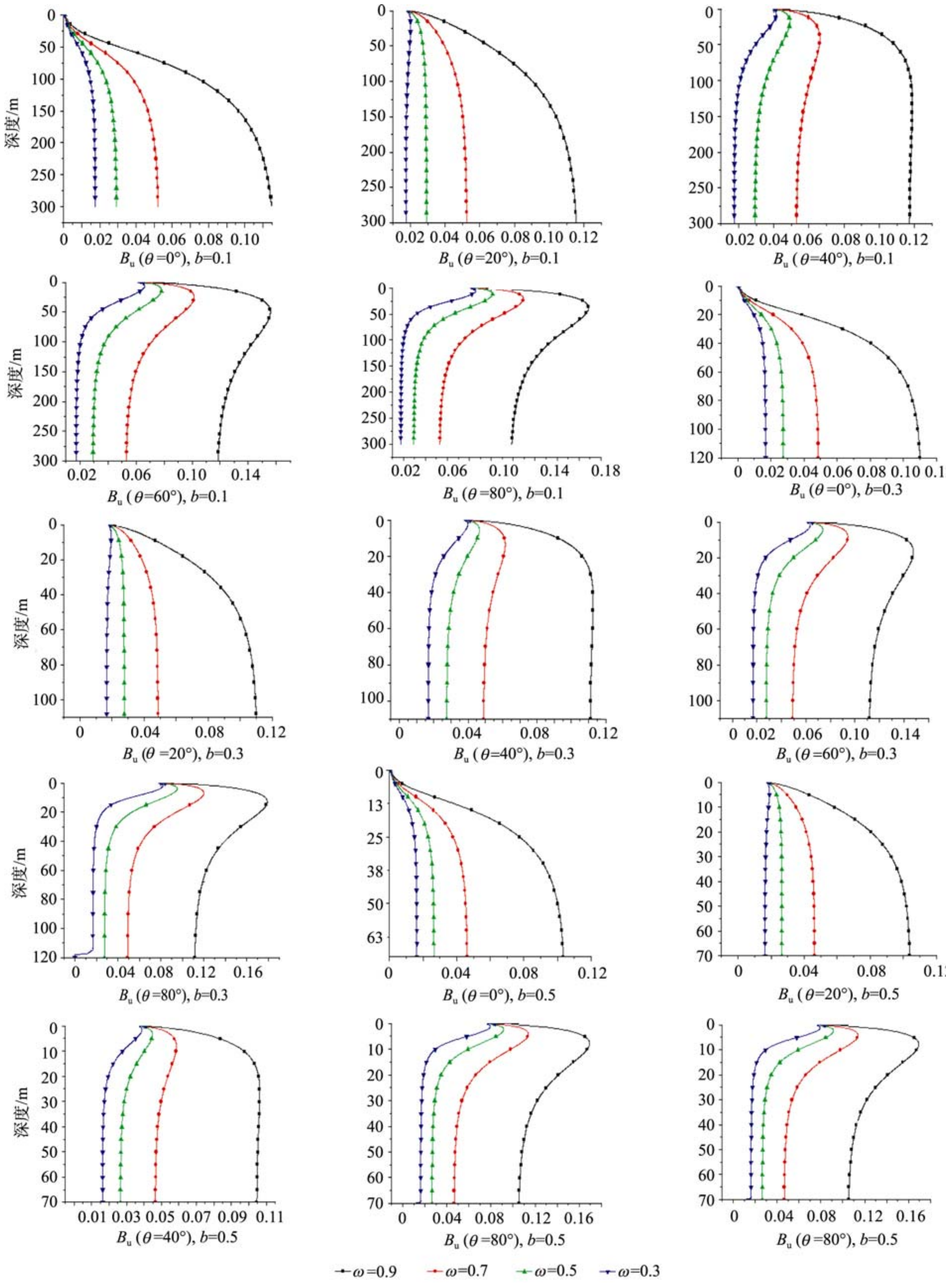

图 2　不同散射系数、单次散射反照率和入射天顶角条件下的向上漫射散射系数的深度廓线

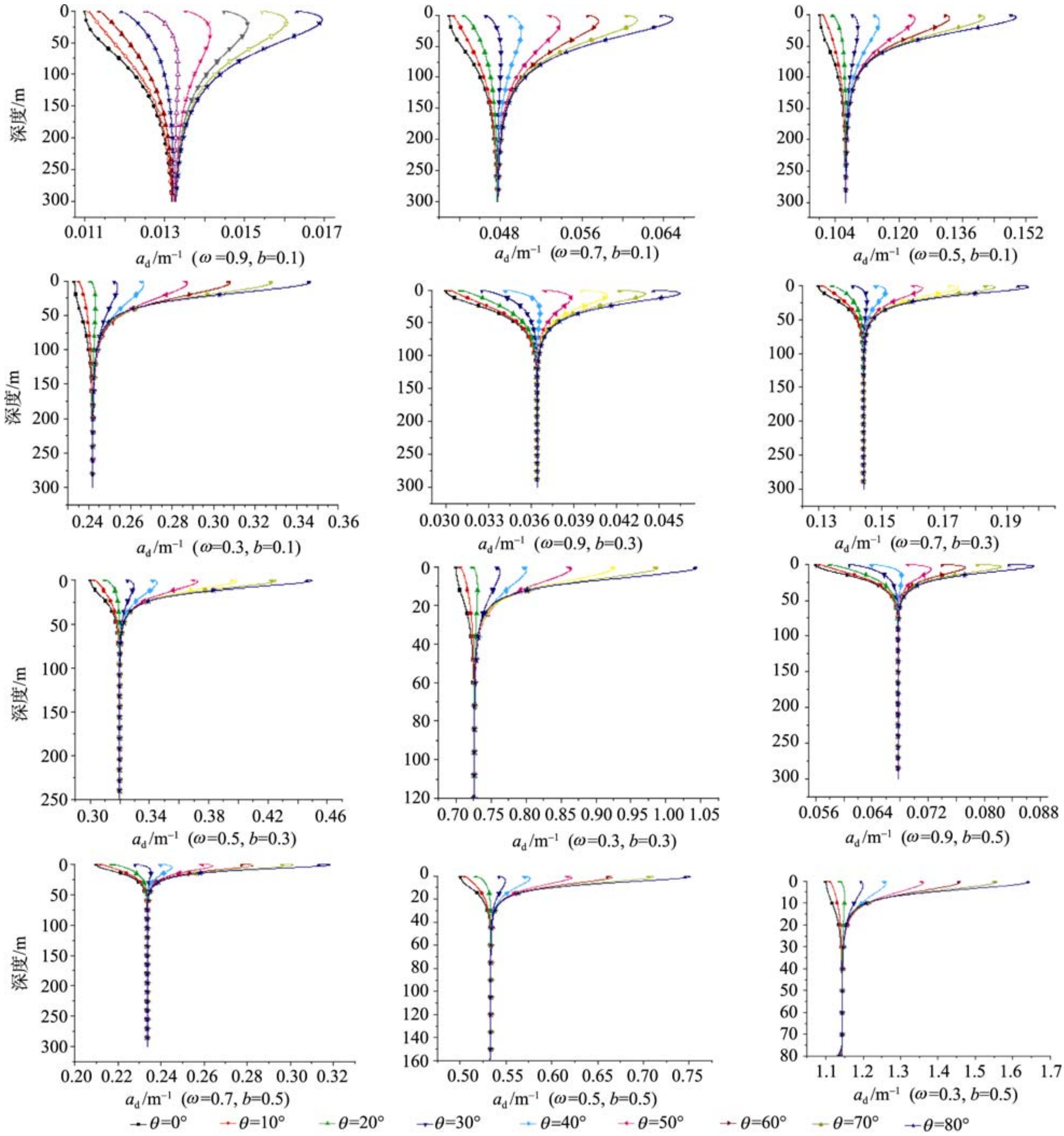

图 3 不同散射系数、单次散射反照率、入射天顶角条件下的下行吸收系数的深度廓线

尽管吸收作用不改变光子传输的方向, 但该过程并非对水下光场的几何结构不产生影响。在水体中, 其吸收作用可表示为(Sathyendranath & Platt, 1991, Kirk, 1999):

$$N = N_0(1-\exp(-aC_w t), \quad (16)$$

式中, N 是被吸收掉光子数, N_0 是初始光子数。C_w 为光在水中的传输速度, t 为光子在水中的传输时间。可见光子在水体中传输的行程越长, 其被吸收的概率也就越大。

随着光子传输时间延长, 光子数被散射的概率和经过的行程加大, 光子被吸收的几率随之增大, 即对天顶角越大的光子, 吸收对其的作用也就越大, 当 2 种作用达到平衡状态时, 即达到渐进光场的状态(Piening & McCormick, 2003)[图 2—4]。由此, 图 2、图 3、图 4 的特征也可以得到较圆满的解释。

在辐射向下传输的过程中, 既不被散射也不被吸收的光子数占入射时光子数的比例随着行程的加大而减小, 沿着辐射初始方向继续传输的光子数越来越少, 当各个方向的光子数基本相当的时候, 也就是渐进态光场出现的时候, 此时许多表观光学参

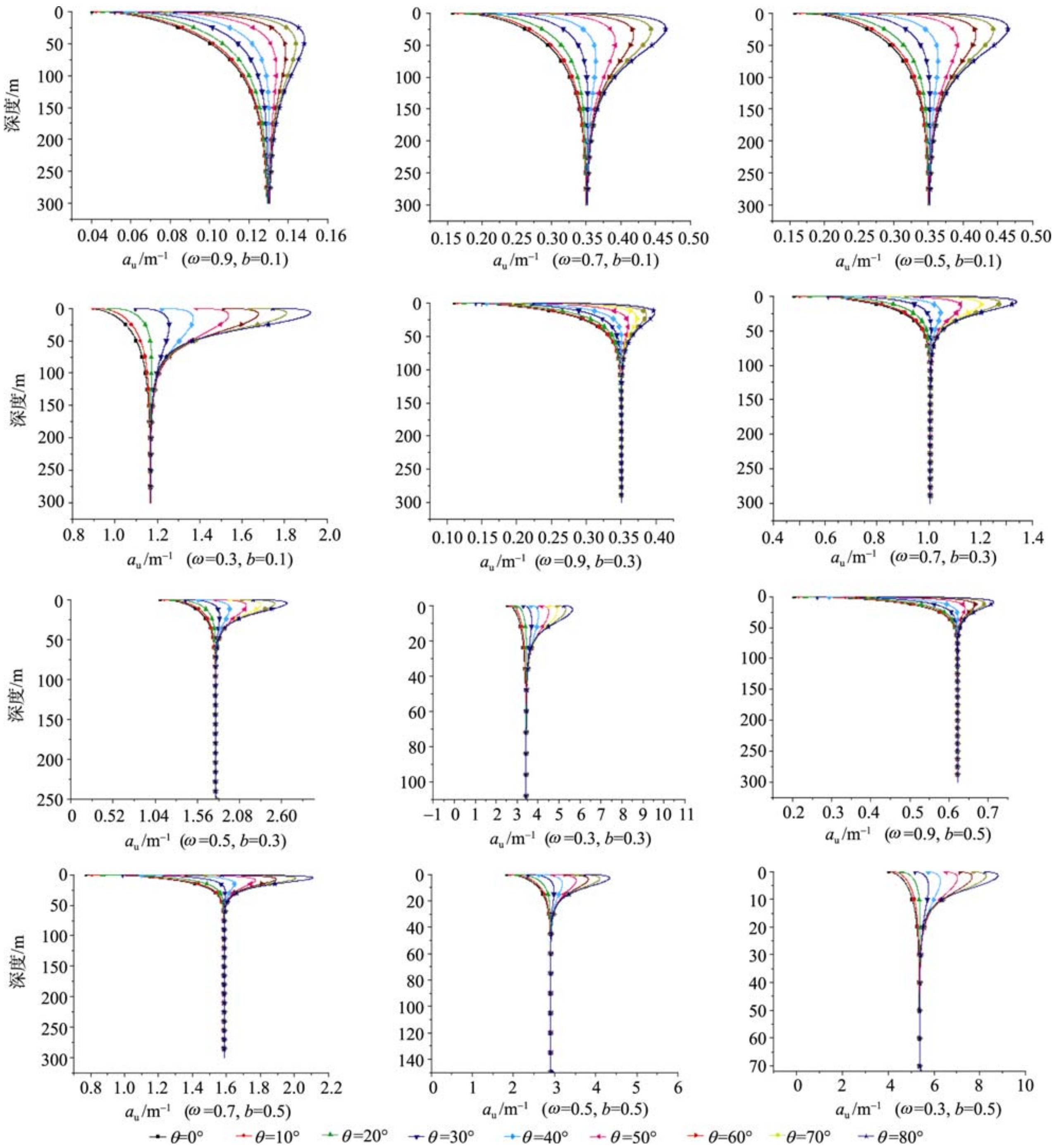

图 4 不同散射系数、单次散射反照率、入射天顶角条件下的上行吸收系数的深度廓线

数也不受诸如水表粗糙程度、大气状况及入射时天顶角的影响，影响他们的仅仅是水体的光学性质，因此这些表观光学参数也就演变成了固有光学参数(Leathers & McCormick, 1997)。

由于向上辐射均是由散射作用引发的，其空间分布完全由介质的体散射相函数决定，所以上行辐射更易被吸收，即上行辐射的漫射衰减系数也就越大，也就使得离水辐射较小。

正由于散射和吸收的共同作用，所以在估算水体初级生产力和光合作用时，必须考虑水下光场的几何结构，不能简单地用固有光学参数和下行辐照度加以计算。

5 结 论

在紧贴水表的下层，向上漫射散射系数随入射辐射天顶角的增大而增大，不随单次散射反照率和散射系数的改变而改变；其深度廓线随深度的增加

逐渐逼近于一恒定的值，该值随单次散射反照率的增大而增大；在入射辐射为垂直入射时，向上漫射散射系数的深度廓线随深度呈现单调增的趋势，并逐步进入一个恒定状态。然而随着入射天顶角的增大，该单调现象发生了变化，出现先增大后减小，再逼进恒定状态，且这个恒定值随单次反照率增大而增大，但不随入射天顶角变化。

下行漫射吸收系数随着入射天顶角的增大，廓线由单调增加过渡到先增大后减小；在散射系数和单次散射反照率相同的条件下，不同入射天顶角的下行漫射吸收系数逐渐收敛于一稳定的状态。且单次散射率越小，这种收敛的速度越快，达到恒定状态的深度也越浅。上行漫射吸收系数的变化特征基本与下行漫射吸收系数类同，只是其值均大于下行漫射吸收系数，其中向上漫射散射系数最大。

REFERENCES

Arst H, Nõges T, Nõges P and Paavel B. 2008. Relation of phytoplankton in stiu primary production, chlorophyll concentration and underwater irradiance in turbid lakes. *Hydrobiologia*, **599**: 169—176

Bannister T T. 1992. Model of the mean cosine of underwater radiance and estimate of underwater scalar irradiance. *Limnol. Oceanogr*, **37**(4): 773—780

Chami M and Robilliard D. 2002. Inversion of oceanic constitute in case I and II waters with genetic programming algorithms. *Applied Optics*, **41**(30): 6260—6274

Elisabeth H S, Duin V, Bloom G, Robert Mafione, Richard Zimmerman, Carl F Cerco, Mark Dortch and Elly P.H. Best. 2001. Modeling underwater light climate in relation to sedimentation, resuspension, water quality and autotrophic growth. *Hydrobiologia*, **444**: 25—42

Gordon H R and McGlunev W R. 1975. Estimation of the depth of sunlight penetration in the sea for remote sensing. *Applied Optics*, **14**: 413—416

Kirk J T O. 1981. Monte Carlo study of the nature of the underwater light field in,and the relationships between optical properties of, turbid yellow waters. *Aust.J.Mar.Freshwater Res*, **32**: 517—532

Kirk J T O. 1989. The upwelling light stream in natural waters. *Limnol. Oceanogr*, **34**(8): 1410—1425

Kirk J T O. 1999. Multiple scattering of a photon flux: implications for the integral average cosine of the underwater light field. *Applied Optics*, **38**(15): 3134—3140

Kirk J T O. 2007. Radiation transfer in ocean water: Average penetration depth of a photon flux in a fixed time interval. *Journal of Geophysical Research*, c02010, doi: 10. 1029/2006JC003500

Leathers R A. and McCormick N J. 1997. Ocean inherent optical property estimation from irradiances. *Applied Optics*, **36**(33): 8685—8698

Liu C C. 2006. Fast and accurate model of underwater scalar irradiance for stratified case 2 waters. *Optics Express*, **14**(5): 1703—1719

Molen D T V D, Los F J and Ballegooigen L V and Vat M V D. 1994. Mathematical modeling as a tool for management in eutrophication aontrol of shallow lakes. *Hydrobiologia*, **275/ 276**: 479—492

Piening B D and McCormick N J. 2003. Asymptotic optical depths in source-free ocean waters. *Applied Optics*, 42, 5382—5387

Ragni M and D'alcalà M B. 2004. Light as an information carrier underwater, *Journal of Plankton Research*, **26**(4): 433—443

Sathyendranath S and Platt T. 1989. Computation of aquatic primary production: Extended formalism to include effect of angular and spectral distribution of light. *Limnol. Oeanogr*, **34**(1): 188—198

Sathyendranath S and Platt T. 1991. Angular distribution of the submarine light field: modification by multiple scattering. *Pro. R. Soc. Lond. A*, **433**: 287—297

Woźnika B, Dera J, Ficek D, Ostrowska M and Majchrowski R. 2002. Dependence of the photosynthesis quantum yield in ocean on environmental factors, *Oceanologia*, **44**(4): 439—459

Woźnika B, Dera J, Ficek D, Majchrowski R, Ostrowska M and Kaczmarek S. 2003. Modeling light and photosynthesis in the marine environment. *Oceanologia*, **45**(1): 171—245