



几种模型在平面坐标转换中的应用

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摘要: 针对在平面坐标转换时选取不同模型将得到不同的转换精度, 通过所编程序进行实验, 对比了四参数模型、六参数模型和二次曲面模型的转换精度。实验表明, 当合理选择转换点时二次曲面模型在平面坐标转换中精度高于四参数模型和六参数模型的转换精度。

关键词: 平面坐标; 坐标转换; 转换模型; 转换精度

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随着测绘技术的发展, 从常规的地面测量发展到卫星大地测量, 在不同时期、不同地方获得了许多基于不同坐标系统的测量成果。由于地图坐标是从参考椭球经过投影转换为平面坐标, 2个不同参考系统之间没有直接的数学关系^[1]。此时, 常常采用二维转换模型进行坐标转换。对于二维转换模型, 参数的选取依赖于工程项目的需要, 在大多数平面坐标转换应用中, 常常使用四参数模型、六参数模型和二次曲面模型进行二维平面坐标转换。本文将对比四参数模型、六参数模型和二次曲面模型在平面坐标转换中的转换精度, 得出算例的最佳模型。

1 模型及其参数求解

1.1 四参数模型

四参数模型^[2]是从布尔莎公式演化而来的, 其计算公式为:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} + (1-m) \cdot \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} \quad (1)$$

式中, Δx 、 Δy 、 θ 、 m 分别为平面上的平移、旋转、尺度参数。不难看出, 要求出 4 个参数至少需要 2 个已知公共点。当有 2 个以上转换公共点时, 将此模型转换为线性模型用最小二乘求解:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c & d \\ -d & c \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} \quad (2)$$

式中, $a=\Delta x$, $b=\Delta y$, $c=m \cos \theta$, $d=m \sin \theta$, $m=\sqrt{c^2+d^2}$ 。

对于 n 个公共点可列如下误差方程:

设所有转换点带有误差的观测值等权, 则由式 (3) 的误差方程, 通过间接平差法求得转换参数向量 \hat{x} 的最小二乘解为: $\hat{x} = (B^T P B)^{-1} B^T P L$ (其中 P 为单位权), 从而求出 \hat{a} 、 \hat{b} 、 \hat{c} 、 \hat{d} 。则平移参数为 $\Delta x = \hat{a}$, $\Delta y = \hat{b}$, 再

$$\begin{bmatrix} V_{X_1} \\ V_{Y_1} \\ V_{X_2} \\ V_{Y_2} \\ \vdots \\ V_{X_n} \\ V_{Y_n} \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_1 & y_1 \\ 0 & 1 & y_1 & -x_1 \\ 1 & 0 & x_2 & y_2 \\ 0 & 1 & y_2 & -x_2 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & x_n & y_n \\ 0 & 1 & y_n & -x_n \end{bmatrix} \times \begin{bmatrix} \hat{a} \\ \hat{b} \\ \hat{c} \\ \hat{d} \end{bmatrix} - \begin{bmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \\ \vdots \\ X_n \\ Y_n \end{bmatrix} \quad (3)$$

用以下两式计算旋转参数 θ 和尺度因子 m : $\theta = \sqrt{\frac{\hat{c}^2 + \hat{d}^2}{\hat{c}^2 + \hat{d}^2}}$, $m = \arctan(-\hat{c}/\hat{d})$

1.2 六参数模型

六参数模型^[1,3]是一种平面仿射变换, 将两坐标轴的指向经过 2 个角度旋转 α 和 β , 并采用 2 个尺度因子, 即纵向尺度因子 λ_x 和横向尺度因子 λ_y 。在任意 2 个平面坐标间的六参数仿射变换, 可以用如下公式:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \lambda_x \cos \alpha & -\lambda_y \sin \beta \\ \lambda_x \sin \alpha & \lambda_y \cos \beta \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \quad (4)$$

式中, Δx 、 Δy 为平移参数; α 、 β 为旋转参数; λ_x 、 λ_y 为比例因子。显然, 要求解 6 个转换参数, 至少需要 3 个公共点的坐标。当有 3 个以上转换公共点时, 可用最小二乘求解转换参数。

将式 (4) 矩阵运算解开, 可得

$$X = \lambda_x \cos \alpha \cdot x - \lambda_y \sin \beta \cdot y + \Delta x$$

$$Y = \lambda_x \sin \alpha \cdot x + \lambda_y \cos \beta \cdot y + \Delta y$$

令 $a_0 = \Delta x$, $a_1 = \lambda_x \cos \alpha$, $a_2 = \lambda_y \sin \beta$

$b_0 = \Delta y$, $b_1 = \lambda_x \sin \alpha$, $b_2 = \lambda_y \cos \beta$

则上式可以写为: $X = a_0 + a_1 \cdot x + a_2 \cdot y$

$$Y = b_0 + b_1 \cdot x + b_2 \cdot y$$

由以上两式可见, X 和 x 、 y 之间存在线性关系,

Y 和 x、y 之间也存在线性关系,因此以上两式完全可以由线性回归原理进行解算。

用一个通用型线性回归模型代替以上两个式子,即 $Z = k_1 + k_2 \cdot x + k_3 \cdot y$ (5)

按最小二乘原理,令(5)式

$$Q = \sum_{i=1}^n [Z_i - (k_1 + k_2 \cdot x_i + k_3 \cdot y_i)]^2 = \min$$

可得线性回归系数 k_j ($j=1,2,3$), 即

$$\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = (B^T B)^{-1} B^T \begin{bmatrix} Z_1 \\ Z_2 \\ \vdots \\ Z_n \end{bmatrix} \quad \text{式中 } B = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ \vdots & \vdots & \vdots \\ 1 & x_n & y_n \end{bmatrix}$$

对于通用线性回归模型(5)将 Z 变为 X, 则 k_1 、 k_2 、 k_3 即分别为 a_0 、 a_1 、 a_2 ; 将 Z 变为 Y, 则分别为 b_0 、 b_1 、 b_2 。

根据 a_0 、 a_1 、 a_2 、 b_0 、 b_1 、 b_2 可以计算出 6 个转换参数: Δx 、 Δy 平移参数, α 、 β 旋转参数, λ_x 、 λ_y 尺度因子。方法如下:

平移参数: $\Delta x = a_0$, $\Delta y = b_0$

由 $\frac{b_1}{a_1} = \tan \alpha$, $-\frac{a_2}{b_2} = \tan \beta$, 则旋转参数:

$$\alpha = \arctan(b_1/a_1), \quad \beta = \arctan(-a_2/b_2)$$

由 $a_1 = \lambda_x \cos \alpha$, $b_1 = \lambda_x \sin \alpha$ 和 $a_2 = -\lambda_y \sin \beta$,

$$b_2 = \lambda_y \cos \beta, \text{ 则尺度因子为: } \lambda_x = \frac{a_1}{\cos \alpha} \text{ 或 } \lambda_x = \frac{b_1}{\sin \alpha},$$

$$\lambda_y = -\frac{a_2}{\sin \beta} \text{ 或 } \lambda_y = \frac{b_2}{\cos \beta}$$

1.3 二次曲面模型

二次曲面模型^[3,4]又称二次多项式拟合,是多项式

变换中的一种,已知点在两坐标系间的坐标差异值可以用一个多项式来拟合,然后用这个多项式来预计其他点的坐标差异值。在坐标系转换中,通常采用二次曲面模型进行拟合。其计算公式如下:

$$\begin{cases} X = a_0 + a_1x + a_2y + a_3x^2 + a_4xy + a_5y^2 \\ Y = b_0 + b_1x + b_2y + b_3x^2 + b_4xy + b_5y^2 \end{cases} \quad (6)$$

式中, a_0 、 a_1 、 a_2 、 a_3 、 a_4 、 a_5 、 b_0 、 b_1 、 b_2 、 b_3 、 b_4 、 b_5 为转换参数,当转换点个数多于 6 个时,可以通过最小二乘法结合线性回归原理求解转换参数,其求解过程与六参数模型参数解算相似。

2 算例分析

所用算例数据^[5]共 15 个公共点,其中 10 个点作为转换点,用于求解转换参数,5 个点作为检验点,点位分布如图 1 所示。为了对比四参数模型、六参数模型和二次曲面模型在两平面坐标系的转换精度,本文采用笔者利用 VB 所编坐标转换程序进行坐标转换,并进行精度分析。

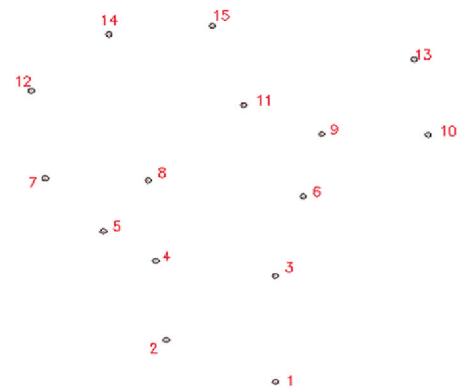


图 1 公共点点位分布图

表 1 三种模型的转换残差比较/m

点号	旧坐标系		新坐标系		四参数模型		六参数模型		二次曲面模型	
	V_x	V_y	V_x	V_y	V_x	V_y	V_x	V_y	V_x	V_y
*1	**2665.804	**9922.330	**2595.126	**9804.030	0.011	-0.008	0.003	-0.006	0.004	0.000
*2	**3644.274	**8855.571	**3573.545	**8737.175	0.002	0.007	-0.004	0.006	0.000	0.004
3	**2594.641	**2535.467	**2523.868	**2417.176	0.000	0.005	-0.004	0.006	0.004	0.002
4	**1819.903	**5699.184	**1749.096	**5580.794	-0.001	0.009	-0.005	0.007	0.004	0.001
5	**2702.922	**1968.380	**2632.076	**1849.953	0.000	0.009	-0.003	0.006	0.002	0.002
*6	**7492.392	**9453.204	**7421.548	**9334.954	-0.004	0.003	-0.005	0.006	0.005	-0.001
*7	**2596.534	**3309.615	**2525.619	**3191.160	0.006	0.001	0.005	-0.005	0.002	-0.001
*8	**0473.752	**2843.572	**0402.868	**2725.190	-0.006	0.011	-0.006	0.009	0.003	0.001
9	**0713.107	**2666.380	**0642.202	**2548.164	-0.002	-0.003	-0.001	0.000	0.009	-0.006
*10	**9258.339	**2482.017	**9187.448	**2363.895	0.003	-0.010	0.004	-0.003	0.000	0.005
11	**7097.886	**8826.439	**7026.946	**8708.156	-0.009	0.006	-0.007	0.007	0.007	-0.004
*12	**0142.466	**1845.588	**0071.459	**1727.145	0.008	-0.005	0.011	-0.011	0.005	-0.002
*13	**6823.344	**8546.181	**6752.376	**8428.065	0.002	-0.012	0.006	-0.005	0.006	-0.001
*14	**3645.083	**3890.272	**3574.050	**3771.893	-0.009	0.006	-0.005	0.003	0.000	0.003
*15	**1684.475	**5725.190	**1613.455	**5606.897	-0.013	0.007	-0.009	0.007	0.004	-0.001

注:表中带*号的点为参与求解转换参数的转换点,其余点为检验点。

表 2 转换残差统计表/m

模型	四参数模型		六参数模型		二次曲面模型	
最大残差	0.011		0.011		0.009	
最小残差	-0.013		-0.011		-0.006	
坐标分量中误差	M_x	M_y	M_x	M_y	M_x	M_y
内符合精度	± 0.008	± 0.008	± 0.006	± 0.006	± 0.003	± 0.003
	$M = \pm 0.011$		$M = \pm 0.009$		$M = \pm 0.004$	
外符合精度	± 0.004	± 0.007	± 0.005	± 0.006	± 0.005	± 0.003
	$M = \pm 0.008$		$M = \pm 0.008$		$M = \pm 0.006$	

转换残差计算公式： $v_x = x' - x$ ， $v_y = y' - y$

其中 x' 、 y' 为转换后坐标值， x 、 y 为同名点原坐标值。

坐标分量 X、Y 转换中误差：

$$M_x = \pm \sqrt{\frac{[v_x v_x]}{n}} \quad M_y = \pm \sqrt{\frac{[v_y v_y]}{n}}$$

当计算内符合精度， n 为转换点个数；计算外符合精度， n 为检验点个数。

$$\text{点位中误差：} M = \pm \sqrt{M_x^2 + M_y^2}$$

从表 1、表 2 可以看出，四参数模型和六参数模型的最大、最小转换残差绝对值都在 1 cm 左右，六参数模型的内符合精度优于四参数模型，外符合精度与四参数模型相同。二次曲面模型的转换精度均高于四参数模型和六参数模型的转换精度，其最大、最小转换残差绝对值都在 mm 级，转换点内、外符合点位中误差也达到了 mm 级。

3 结 语

本文通过对比 3 种模型在平面坐标转换中的转换精度可以看出，当选择合理的转换点点位和转换点个数进行转换参数的求解时，3 种模型的转换精度都在

cm 级范围内，由于六参数模型采用了 2 个旋转参数和 2 个尺度因子，其转换精度比四参数模型的转换精度高，而二次曲面模型由于考虑了系统畸变的影响，在 2 种转换模型中转换精度最高，达到 mm 级。

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4 结 语

根据本文所采用的转换公式能满足生产上的应用，具有实际应用价值。高斯正反算用计算机编程计算其结果是有一定误差的。

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Abstract This paper introduced the concept of common height system in the measurement process, GPS elevation fitting method and scope of application. It analyzed and discussed the method of rapid change in the height anomaly area to meet the GPS Fitting Height with 1: 10 000 precision control requirements.

Key words height system, GPS, aerial photogrammetry, adjustment, error
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Research of CORS Station Stability Monitor Based on Gamit
by ZHANG Xudong

Abstract CORS has founded and been maintaining the regional control survey frame and benchmark with multi-station, whose stability are very important to the system. Because of the distances between stations are larger than 40km, common GPS data processing software can not process the data with high precision. Taking Ningbo CORS as example, this paper studied on monitoring the CORS stations' stability with Gamit data processing.

Key words GAMIT, CORS station, stability monitoring
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Construction of TIN and Generation of Contour Line on AutoCAD
by DAI Li

Abstract The generation of TIN was being analysed. According to algorithm of triangle generation, construct the TIN while based on discrete point in AutoCAD, and generated contour line of arbitrary height the same.

Keywords Digital Elevation Model, Triangular Irregular Network, Delaunay triangular network, contour line
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Application Research of Geographic Information Platform for Public Emergency Services in Hubei Province
by NIE Xiaobo

Abstract Summary of the provincial emergency response system, the basic geographic information platform needs, explained how digital space-based information infrastructure, used of the network geographic information system technology (WebGIS) to integrate basic geographic information resources and the resources of public emergency project to build provincial Public Emergency Services Geographic Information Platform's overall design and technical implementation.

Key words emergency platform, public emergency incident, WebGIS, public safety
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Application of AutoCAD and Surfer to the Contour Drawing
by ZHAO Fang

Abstract This paper described the significance of contour and the principle of Surfer drawing. Details of the use of Surfer and AutoCAD combined contour drawing methods and procedures. Because of its accuracy and rapidity, it improved the graphics quality and efficiency.

Key words Surfer, AutoCAD, contour, coal mine, Map
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Design and Application of Decision Support System for Negotiation and Delimitation of National Boundaries
by LIU Hehui

Abstract The negotiation and delimitation of national boundaries is an important and complicated problem. This paper discussed and introduced the design of functional modules and the system data design based on the spatial analysis technology of GIS, after analyzing the business process of the negotiation and delimitation of national boundaries. And this system could effectively manage data, and provided the tools for auxiliary delimitation and resources evaluation. These provide effectual support for the negotiation and delimitation.

Key words negotiations and demarcation of national boundaries, ArcGIS Engine, documents directory tree, auxiliary demarcation, document database
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Design of Drawing Documents Information Management System Based on ArcIMS
by WANG Xianpu

Abstract This paper researched design and development of drawing documents information management system, introduced ArcIMS and this system framework, functional design, database design, especially introduced attribute table design.

Key words drawing documents sharing, ArcIMS, database design
(Page:52)

Accuracy of Real-time Range Assessment for CORS
by LI Bo

Abstract An assessment method for the accuracy of near real-time range was proposed based on the pseudo range observation equation and the character of the CORS stations, and then the key problems of that were expounded in detail. In addition, the performance and adaptively were demonstrated base on real GPS data and the result gave the confidences that the assessment could be used in practical successfully.

Key words CNSS, CORS, accuracy
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Establishment Website Dedicated on Government Administration GIS Plat-

form by WANG Yiqin

Abstract Taking the Subject of Geographic Information System (GIS) on fundamental realities of Yunnan under the program of "Public Access Spatial Information Platform on South Asian Association for Regional Cooperation (SAARC) in South East Asia regional cooperating China (Yunnan)-EASAN Free Trade Zone as a case, this paper introduced methodologies on home page development of GIS platform, including page layout, information structuring, map service call up. Methods for web page items control as well as its application prospect were discussed in detail, which hopefully is of reference value for the development and application of similar function for government administration GIS platform.

Key words government administration GIS, web page control, digital information tree, map service
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Transformation Program and Precision Analysis between Geodetic Coordinate and Gauss Plane Coordinate
by XU Lei

Abstract The paper gained formula that adopted computer computation based analyzing transformation formula between geodetic coordinate and Gauss plane coordinate. It adopted method that programs many subprogram and realized transformation between geodetic coordinate and Gauss plane coordinate, programs to realize transformation Beijing 54 coordinate, Xi'an 1980 coordinate, 30 band Gauss plane coordinate and 60 band Gauss plane coordinate. The paper analyzed precision about transformation results, drew a conclusion that it can meet ordinary production use adapting the transformation program, but there was a little error.

Key words Geodetic coordinates, Gaussian Cartesian coordinates, coordinate transformation, precision analysis
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Application of Several Models to Plane Coordinate Transformation
by YAO Chaolong

Abstract Aiming at the coordinate transformation between two 2D coordinate systems, different accuracies from different models will be achieved. This paper utilized the program to compare the accuracy of four-parameter model, six-parameter model and second-degree polynomial model. Results from testing showed when reasonable choose transforming points, the accuracy of second-degree polynomial model is better than the accuracies of four-parameter model and six-parameter model in 2D coordinate transformation.

Key words plane coordinate systems, coordinate transformation, conversion model, transformation accuracy
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Change Detection Based on Aviation Remote Sensing Image
by XV Xiaoqin

Abstract In this paper, the change detection method of utilizing aviation images' grey level difference and ratio was mainly studied. In order to analyse this two kinds of methods, a group of aviation images taking farmland as main landscape of the whole view was measured. According to the result of experiments presented by the form of black-and-white pictures, while relatively choosing different threshold value, these two kinds of methods were compared, especially in the use of detecting the result through the change of this specific goal of farmland, and the combination of the two methods was applied as a final result.

Key words grey level difference, grey level ratio, change detection, aviation image
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Research on Space Road Network of Kaifeng Based on the Model of Space Syntax
by XU Chong

Abstract The model of space syntax analysis is an objective method of analyzing road network. This paper analyzed the accessibility and the spatial distribution of accessibility regional of Kaifeng with the analysis functions of Arcview and ArcGIS. And then discussed the problem of Kaifeng road network and give some improvement measures.

Key words space syntax, space road network, Kaifeng
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Quality Evaluation Methods of Large Scale Digital Map Before Import into Database
by LI Xuanrui

Abstract Firstly, the importance of quality evaluation of large scale digital map before import into database were introduced. Then the mathematical model and the steps of fuzzy mathematics judgement applied in quality evaluation were given. Lastly, an engineering example was given to verify the feasibility of the method, this method can ensure the quality of large scale digital map before import into database.

Key words large scale digital map, quality evaluation, fuzzy mathematics judgement
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L2C Signal and L2 Carrier Data Quality Analysis
by LI Weijun

Abstract Based on the IGS tracking station data, the signal-to-noise ratio (SNR) of L2 carrier phase resumed by L2C code was improved obviously, almost close to the SNR of L1 carrier. For different receivers, this paper followed with a study on the multipath and noise levels of C/A and L2C code pseudorange, pointing out for TRIMBLE